Reserves and the Buyer of Last Resort

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Abstract

We examine the role of central bank reserves and public liquidity when secondary markets may freeze. Central bank reserves help intermediaries purchase assets during stress but crowd out investment. Under laissez-faire, intermediaries hold insufficient reserves, overlooking how aggregate liquidity reduces freeze risk. We propose a "market-backstop principle", akin to Bagehot's principle for intermediaries. It combines state-contingent buyer-of-last-resort interventions to restore trading with modest liquidity requirements to limit moral hazard. The welfare benefits of restoring market functioning exceed the fiscal costs of interventions. We explore implications for the size and composition of central bank balance sheets.

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> "*When the financial system teetered near collapse in 2008 and 2009, we responded as the 19th century British essayist Walter Bagehot advised, by serving as liquidity provider of last resort to stressed financial firms and markets. But we did so in an institutional environment that was very different, and in many ways much more complex, than the one that Bagehot knew.(...) Rather than a run of retail bank deposits, the runs occurred in various forms of short-term, uninsured wholesale funding.*"
>
> Ben S. Bernanke, then Chairman of the Federal Reserve (Bernanke, 2013)

1 Introduction

Modern financial instability often takes the form of freezes in secondary securities markets. Crises need not begin with depositors running on banks; they may begin when some financial institutions are unable or unwilling to absorb the assets that others need to sell. When secondary markets freeze, intermediation breaks down: assets fail to reach their natural holders, and the gains from efficient trade go unrealized. The 2007–08 freeze in the U.S. asset-backed commercial paper market, the global "dash for cash" of March 2020, and the U.K. gilt-market turmoil of autumn 2022 illustrate different facets of this modern form of market dysfunction.

While these episodes differ in their triggers and in the financial assets and institutions involved, they share a common constrained-absorption pattern. A class of financial assets comes under sudden stress — commercial paper, U.S. Treasuries, or U.K. gilts. Some institutions become urgent sellers — money-market funds (MMFs), prime MMFs, or liability-driven investment funds. At the same time, the investors best placed to absorb these assets — commercial banks, primary dealers, or hedge funds — are themselves constrained by balance-sheet capacity, risk limits, or funding conditions. The economic structure is therefore similar across episodes: distressed assets need to move from forced sellers to natural buyers, but forced sales overwhelm the buyers' absorption capacity, as described by Granja et al. (2017); Duffie (2023); Duffie et al. (2023). This is the market-freeze mechanism we have in mind throughout the paper.[1]

Policymakers have responded to such freezes along two margins. The first is *ex ante*: regulators have required banks to hold liquidity buffers before stress materializes, thereby increasing system-wide holdings of public liquidity, notably central bank reserves (Banerjee and Mio, 2018; Kedan and Veghazy, 2021).[2] The second margin is *ex post*: central banks

[1] Table A in the Appendix illustrates how this constrained-absorption mechanism maps into the three market-freeze episodes discussed above.

[2] The main example is Basel III's Liquidity Coverage Ratio (LCR), which requires banks to hold enough high-quality liquid assets to withstand a short period of funding stress. The LCR is primarily designed as a *microprudential* tool to help individual banks withstand liquidity stress and avert runs. At the same time, the Basel Committee also emphasizes a broader systemic rationale and the "importance of liquidity to the proper functioning of financial markets and the banking sector" (BCBS, 2013, introduction). This paper builds on that *macroprudential* dimension, focusing on the role of liquidity buffers — and central

have acted as buyers of last resort (BOLR), purchasing impaired assets with newly issued reserves in order to restore trade, even though they are not necessarily the natural holders of the assets they acquire.[3] Central bank reserves link these two margins. They are the asset issued by public backstops in crises, but also the liquid asset that allows banks to keep buying when markets are strained, reducing both the likelihood and the severity of freezes.

This paper asks how these two uses of reserves should be combined. We derive a market-level analogue of Bagehot's lender-of-last-resort (LOLR) principle,[4] shifting the focus from backstopping individual financial institutions to backstopping wholesale financial markets. We call this the "market backstop principle": *the central bank should intervene only when trade breaks down, purchasing distressed assets only in quantities sufficient to restore market functioning but no larger than necessary, while paying the fair price that prevails once functioning has been restored. A minimum regulatory liquidity buffer, in turn, should address the moral hazard created by the prospect of intervention — no more and no less.* The resulting liquidity requirement is modest: it raises banks' reserve holdings relative to what they would choose under the backstop, but remains below their laissez-faire holdings in the absence of intervention. Although interventions may generate central bank losses in periods of stress, they are welfare-improving when properly calibrated. These losses should therefore be understood as the fiscal cost of restoring market functioning, rather than as a sign of policy failure.[5] At the same time, calibration matters: a poorly designed intervention can reduce welfare relative to laissez-faire.

Our market backstop principle is derived from a model that formalizes the constrained-absorption view of market freezes.

Model in a Nutshell. Central bank reserves are valuable in the model, but costly. They are valuable because they provide liquidity when financial markets freeze; they are

bank reserves in particular — in supporting market functioning.

[3]We focus on asset purchases aimed at restoring market functioning — "market-function" purchases in the terminology of Duffie and Keane (2023). These purchases differ from quantitative easing, which uses asset purchases to stimulate the economy; see Bernanke (2020). In recent crises, central banks have also provided emergency lending to banks. We abstract from this type of interventions here. For detailed documentation on historical market-support programs, see Yale's New Bagehot Project.

[4]The Bagehot principle is often seen as the foundation of the central bank's role as LOLR. It prescribes that, in a crisis, the central bank should lend early and freely to solvent institutions, against good collateral, and at a high, or penalty, rate.

[5]Central banks' large-scale asset purchases may generate profits or losses for several reasons, such as interest-rate exposure, duration, and risk profile of the assets acquired, as well as subsequent valuation changes. Levin et al. (2022), for example, estimate that the Federal Reserve's large-scale asset purchases launched at the onset of the Covid-19 pandemic in March 2020 and concluded in March 2022 could reduce cumulative remittances to the U.S. Treasury by about \$760 billion by 2032 — around 0.2% of GDP per year over that twelve-year period. Later in the paper, we clarify the specific source of the losses considered in our analysis. The point here is that such losses, when they occur, need not signal policy failure; they may instead be part of the fiscal cost of restoring market functioning.

costly because they absorb resources that could otherwise finance productive investment, as shown empirically by Sundaresan and Xiao (2024). To fix ideas, the model features a continuum of banks that initially allocate their funding between reserves and business loans. Banks then learn whether they are "good" or "bad". Good banks monitor the firms underlying these loans efficiently and generate high returns; bad banks monitor them poorly and generate low returns. This heterogeneity creates gains from secondary-market reallocation: business loans generate more value when held by good banks than by bad banks. Bad banks therefore become natural sellers of these assets.

The terms "banks" and "business loans" are meant as generic labels and should be interpreted broadly. Banks stand for *financial intermediaries*. Bad banks represent *natural or forced sellers*, while good banks represent *natural buyers or holders*: investors whose asset-management capacity creates the greatest value added, or whose clients benefit most from holding the assets. Likewise, business loans stand for *financial assets* whose market liquidity — and hence reallocation — may become impaired during a crisis.[6]

In our model, reallocation can occur through two secondary markets. In the cash market, business loans are exchanged for central bank reserves. In the credit market, they are sold against promises of future repayment. The cash market is frictionless because reserves are information-insensitive (Gorton and Pennacchi, 1990; Dang et al., 2017, 2020). The credit market, by contrast, is subject to asymmetric information and moral hazard. Bank types are private information, so prospective lenders cannot identify which banks are the natural holders of business loans. This friction allows bad banks to mimic good banks by borrowing to purchase these loans and subsequently defaulting — a form of search for yield (Martinez-Miera and Repullo, 2017; Boissay et al., 2026). Lenders respond by imposing borrowing limits. When returns fall and business loans become less valuable overall, these limits bind for good banks, leaving them unable to absorb bad banks' forced sales. The credit market freezes, and assets fail to reach their natural holders.[7]

In the laissez-faire equilibrium, banks choose inefficiently low holdings of central bank reserves. Privately, reserves provide self-insurance: if a bank turns out to be good, it can use them to purchase business loans at depressed cash-market prices during a credit-market

[6]These generic labels are useful because the distressed assets and financial institutions involved vary across market freezes, as illustrated in Table A in the Appendix. Beyond technological returns from monitoring business loans, good banks' excess return may also capture, for example, a preferred-habitat motive — see Section A in the online appendix. Some assets create greater value when held by investors naturally suited to hold them, either because these investors manage the assets more efficiently or because their clients derive greater utility from the assets' payoff structure.

[7]The way we model financial-market freezes is akin to Dang et al. (2020)'s information view of financial crises. This view holds that, when fundamentals are strong, lenders are confident that borrowers will repay and therefore lend "without asking questions": lending is information-insensitive. When fundamentals are weak, by contrast, lenders become concerned that some borrowers may default and, lacking information about prospective borrowers, may refuse to lend: lending becomes information-sensitive.

freeze. Socially, aggregate reserves also reduce both the probability and the severity of freezes, an effect that individual banks fail to internalize. This pecuniary externality depresses private reserve demand below its efficient level.

We analyze two policy instruments. The first is liquidity regulation: a minimum liquidity requirement that reduces the probability of a market freeze, but also forces banks to hold low-return reserves instead of financing productive projects. The second is a BOLR backstop: in crisis states, the central bank purchases business loans and restores trade in the credit market. Such interventions raise welfare for any given reserve buffer because they reallocate assets to their natural holders and mitigate the real losses associated with market freezes. Used in isolation, however, a backstop creates moral hazard. If banks expect the central bank to intervene whenever the credit market freezes, they reduce their own precautionary reserve holdings, in the limiting case to zero. Because it induces excessive reliance on *ex post* public support, a market backstop is not always socially desirable.

Contribution. Our primary contribution is to characterize a set of general guidelines for backstopping wholesale markets — the market backstop principle. The principle parallels Bagehot's LOLR doctrine, but shifts the object of protection from individual financial *institutions* to wholesale financial *markets*. It combines an intervention that restores trade in crisis states with a modest liquidity requirement that limits the moral hazard created by the prospect of intervention. The optimal policy mix varies with economic fundamentals. Two forces are central: the expected return on business loans and the expected cost of credit-market freezes. We summarize the latter by *return-at-risk*: the expected return forgone in crisis states when banks enter the crisis without reserves. When business loans have high expected returns, reserves are costly because they crowd out productive lending; optimal policy therefore relies more on *ex post* backstops and less on *ex ante* reserve requirements. When return-at-risk is high, by contrast, freezes are more costly because they are more likely, more severe, or both. In that case, backstops alone become less attractive: banks choose too little liquidity, while the central bank must intervene more often, purchase more assets, and bear larger losses. The optimal policy therefore typically combines both instruments, with a modest liquidity requirement that decreases with the expected return on business loans and increases with return-at-risk.

The analysis also clarifies how reserve management affects the size and riskiness of the central bank balance sheet. A higher reserve requirement mechanically increases the *ex ante* size of the balance sheet. At the same time, it makes crises less frequent and reduces the scale of backstop purchases when crises occur. Once state-contingent backstops are taken into account, a higher reserve requirement may therefore limit crisis-time balance-sheet expansions and leave the central bank with a smaller balance sheet *on average.*

The same logic applies to average central bank losses. Asset purchases are essential for restoring market functioning, but they are costly because the central bank is not the natural holder of the assets it acquires. Because our model is static, these losses do not arise from duration risk or other valuation effects. Rather, they reflect a form of "negative carry": the return earned by the central bank on the assets it purchases is lower than the cost of the reserves issued to finance them. In effect, this negative carry amounts to a subsidy to asset sellers during crises. Losses are therefore tied to purchases of distressed assets in crisis states and to the reserves issued *ex post* to finance those purchases; they are not driven by reserves issued *ex ante*. A larger *ex ante* balance sheet therefore generates smaller losses than a smaller balance sheet that must repeatedly expand through potentially loss-making backstops.

Related Literature. Our paper connects to several strands of the literature on financial intermediation and central banking. First, it builds on the classical literature on liquidity provision and financial crises, notably Diamond and Dybvig (1983) and Holmström and Tirole (1998).[8] Like these contributions, we view the provision of safe, liquid assets as a central public-sector function when private markets cannot generate sufficient inside liquidity. This view is closely related to the classical LOLR doctrine, under which monetary authorities lend freely to solvent but illiquid banks during crises (Bagehot, 1873; Bordo, 2002; Freixas et al., 2004; Rochet and Vives, 2004). In modern financial systems, however, central bank support extends beyond lending to individual institutions: it also encompasses BOLR interventions and asset purchases aimed at stabilizing wholesale markets. For example, Acharya et al. (2021) characterize the European Central Bank as a "buyer of last resort" during the euro-area sovereign debt crisis. Whereas the Bagehot principle addresses liquidity provision to individual institutions, our paper studies the design of backstops for financial markets.[9]

Second, our mechanism builds on the literature on asymmetric information and market breakdowns, starting with Akerlof (1970). The underlying friction is similar to that in Stiglitz and Weiss (1981) and Boissay et al. (2016, 2026), where asymmetric information can prevent credit markets from clearing.[10] We extend this mechanism by introducing a

[8] See also Diamond and Kashyap (2016) and the references therein for a recent discussion of optimal liquidity regulation and liquidity provision to individual banks in a Diamond and Dybvig (1983) framework.

[9] Our focus also differs from that of Hong et al. (2008), who study how non-financial firms can act as "buyers of last resort" of their own stocks in distressed markets.

[10] A similar approach to introducing financial frictions in wholesale secondary markets can be found in Boissay and Cooper (2020); Dong and Xu (2020); Garcia-Macia and Villacorta (2023).

cash market in which central bank reserves serve as an interbank settlement asset immune to information frictions. In this respect, our analysis also builds on Gorton and Pennacchi (1990) and Dang et al. (2020), who emphasize the role of information-insensitive assets in facilitating trade.

Third, the paper contributes to the growing literature on the effects and transmission channels of BOLR interventions, central bank asset purchases, and state-contingent policy commitments. On the empirical side, Duffie and Keane (2023) and D'Amico et al. (2026) distinguish between asset purchases aimed at restoring market functioning and those aimed at stimulating the broader economy. Duygan-Bump et al. (2013), D'Amico and King (2013), Fleming et al. (2021), Kargar et al. (2021), and O'Hara and Zhou (2021) show that Federal Reserve asset purchases helped restore secondary-market functioning during the Global Financial Crisis and the Covid-19 pandemic. Duffie et al. (2023) emphasize that occasionally binding borrowing constraints faced by natural buyers are an important source of Treasury-market dysfunction. Haddad et al. (2025) document that expectations of market backstops — "policy puts" — account for a large share of the effects of large-scale asset-purchase announcements. Breckenfelder and Schepens (2025) study the ECB's corporate debt purchases during the 2020 market freeze and show that, while such interventions alleviated immediate liquidity constraints, they also generated longer-run funding distortions. On the theoretical side, Bianchi and Bigio (2022) model central bank reserves as a means of meeting liquidity needs in the interbank market, while Roch and Uhlig (2018) and De Fiore et al. (forthcoming) study related crisis interventions through bailouts and haircut policies. Closest to our paper, Choi and Yorulmazer (2024) and Kahn et al. (2025) analyze BOLR interventions in environments with multiple equilibria, but abstract either from moral hazard or from the policy tools that may be used to contain it. We extend this literature by characterizing the optimal policy mix for market backstops: state-contingent asset purchases at restored-market prices, combined with a modest minimum liquidity requirement. In this respect, we also connect to Magill et al. (2020) and Aramonte et al. (2023), who call for public liquidity backstops to be accompanied by *ex ante* regulatory constraints to contain moral hazard.

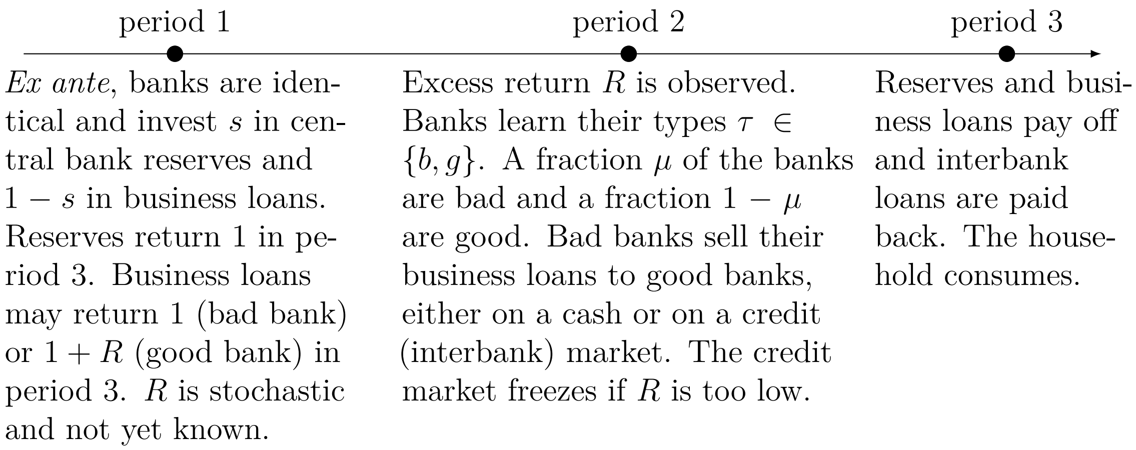
Finally, our analysis speaks to the recent debate on the adequate size of central bank balance sheets and the role of reserves in financial stability. Stein (2012) and Greenwood et al. (2015) emphasize the stabilizing benefits of central bank reserve provision, while more recent work highlights the costs and distortions that may arise when reserves are abundant. In particular, Acharya et al. (2022) and Acharya and Rajan (2024) document a form of "liquidity dependence", whereby abundant reserves induce banks to expand demandable liabilities and credit lines, increasing the financial system's reliance on central bank support. We contribute to this debate by highlighting the substitutability between banks' *ex ante* demand for reserves as self-insurance and the central bank's *ex post* supply of reserves through market backstops.[11]

The paper proceeds as follows. Sections 2 and 3 present the model and characterize the laissez-faire equilibrium. Section 4 derives the optimal mix of liquidity regulation and BOLR backstops. Sections 5 and 6 discuss the policy trade-offs, their dependence on fundamentals, and their implications for the size of the central bank balance sheet and expected losses. The final section concludes.

2 The Model

We consider an economy with three periods: 1, 2, and 3. The economy is populated by a central bank and a continuum of each of households, banks, and firms. The model is formulated in real terms. Figure 1 provides a timeline and an overview of the model. We now describe the environment in detail.

Figure 1: Timeline



2.1 Endowments, Preferences, Technologies

Each **household** is endowed with one unit of an investment good in period 1. Households consume in period 3 and are risk-neutral.

There are two **technologies**: storage and projects. Storage is available to firms, banks, and the central bank, but not to households. It transforms one unit of the investment good in period 1 into one unit of the consumption good in period 3. Projects, by contrast, can be operated only by **firms** and are financed by bank business loans. In the spirit of Diamond (1984)'s theory of delegated monitoring, projects require supervision, or

[11] Keister (2016) and Jeanne and Korinek (2020) also study the trade-off between *ex ante* and *ex post* policies, but focus on different forms of *ex post* intervention, such as bailouts or LOLR.

monitoring, by banks. **Monitoring** can be either "good" or "bad". If monitoring is bad or absent, a project is equivalent to storage: it transforms one unit of the investment good in period 1 into one unit of the consumption good in period 3. If monitoring is good, the project instead yields $1 + R$ units of the consumption good in period 3, where $R \in \mathbb{R}^+$. We assume that the excess return R is an aggregate random variable realized in period 2, distributed according to the cumulative distribution function F, with strictly positive density f on $\mathbb{R}^+$.[12] Its expected value is denoted by $E(R) = \rho$.

Banks are identical *ex ante* and issue deposits in period 1. Deposits are denominated in units of the investment good, so that one unit of deposits corresponds to one unit of the investment good. Banks may invest these resources in storage, central bank reserves, or loans to firms, which correspond to investments in firms' projects. For simplicity, we normalize each bank's deposits to one.[13] Let α denote the fraction invested in business loans. We assume that originating business loans entails resource costs, for instance due to project screening, paid in period 3 by the originating bank and equal to

$$\phi = \kappa\alpha \tag{1}$$

in units of the consumption good, where κ is a parameter satisfying $0 < \kappa < \rho$.[14] In period 2, a fraction $1 - \mu$ of banks are revealed to have the capacity for good monitoring. We refer to these banks as "good" banks, and to the remaining fraction μ as "bad" banks.[15]

The **central bank** can operate the storage technology and issue reserves. We model reserves as claims on the central bank that promise one unit of the consumption good in period 3 and can be used in period 2 as an interbank settlement instrument, or means of payment. Reserves are the only such instrument.[16]

2.2 Markets

Banks and firms are owned by households. Their shares are not traded.

Period 1: In period 1, banks extend loans to firms in the form of bank deposits, which firms use to purchase investment goods. In exchange, banks receive claims on firms'

[12]Our analysis carries through if the domain of f is restricted to an interval $[\underline{R}, \overline{R}] \subset \mathbb{R}^+$.

[13]Although a given household is not tied to a particular bank and banks compete for households' deposits, in a symmetric equilibrium all banks issue the same amount of deposits and hold the same share of households' initial endowment of investment goods.

[14]If $\kappa > \rho$, investing in loans is never worthwhile.

[15]Hence, $1 - \mu$ captures the size of the natural-buyer base. The higher μ, the more limited the set of natural holders of business loans.

[16]One may ask why this role is assigned exclusively to the central bank rather than also to the private sector. We view this assumption as a reasonable description of the current functioning of the banking system. It is analogous to the standard assumption that only the central bank can issue fiat money. The reasonableness of that assumption may similarly be debated in light of the recent emergence of cryptocurrencies.

period-3 project payoffs. While both sides are in perfect competition, banks also affect loan returns by monitoring projects in period 2. We therefore assume that banks have all the bargaining power, and that firms pay the entire payoff of their projects to the bank that owns the loan in period 3. Thus, per unit of loan owned, a good bank receives $1 + R$ units of the consumption good in period 3, while a bad bank receives 1. Loan-origination costs, $\phi = \kappa\alpha$, are paid separately by the bank that originated the loans.

Firms use their loans to purchase investment goods from households, who in turn hold the deposits they receive at a bank. We assume that banks offer deposit contracts to households in Bertrand competition, promising to pay $1 + \gamma$ units of the consumption good in period 3 per unit of deposits. Hence, banks break even on their deposit contracts.[17] The deposit return γ is endogenous and determined in period 1.

Banks may sell to the central bank any investment goods that are not invested in business loans. We assume that the central bank fixes the price of one unit of the investment good at one unit of reserves. The central bank then stores the purchased investment goods until period 3. Since each stored investment good yields one unit of the consumption good in period 3, and since central bank reserves promise one unit of the consumption good in that same period, the central bank breaks even on this transaction.[18] This trade can be interpreted as an abstraction of a richer institutional arrangement in two ways. First, banks may be viewed as storing the investment goods themselves and selling ownership claims on the stored goods to the central bank. Second, one may introduce a treasury that supplies government securities on demand to banks. Banks may then either hold these securities or sell them to the central bank for central bank reserves through an open-market operation. These alternative interpretations are immaterial for the analysis, and we therefore abstract from them.

Since stored investment goods yield the same return as reserves but, unlike reserves, cannot be used as an interbank settlement instrument in period 2, banks weakly prefer to sell these goods to the central bank and hold reserves rather than retain

[17]Households may be viewed as having all the bargaining power vis-à-vis banks and extracting the entire surplus, so that banks break even on deposits. Since households own the banks and are risk-neutral, this assumption is not essential, but it allows us to introduce a profit-maximization motive for banks in a clean way.

[18]One could depart from this assumption by allowing the central bank to offer either more or less than one unit of reserves, thereby generating central bank losses or gains. Although this extension is interesting in its own right, our focus here is on liquidity regulation and BOLR interventions. We therefore do not pursue it further.

them on their balance sheets.[19] Hence, the $1 - \alpha$ stored goods remaining on banks' balance sheets are sold to the central bank. Denoting banks' reserve holdings by s, we have

$$s = 1 - \alpha. \tag{2}$$

A **minimum regulatory liquidity requirement** s_r may be in place, so that banks must choose s satisfying

$$s \geq s_r. \tag{3}$$

Period 2: The excess return R, common to all business loans subject to good monitoring, is realized in period 2. Recall that the return is paid in period 3. At the same time, idiosyncratic bank types $\tau \in \{b, g\}$ are revealed: a fraction μ of banks are bad, $\tau = b$, and a fraction $1 - \mu$ are good, $\tau = g$.

There are two interbank markets. Both can be interpreted as secondary markets for business loans. The first is a loan-for-credit market, or **credit market**, in which one unit of business loans is traded against a promise to pay $1 + r$ units of the consumption good in period 3.

The second is a loan-for-reserves market, or **cash market**, in which one unit of business loans is traded for q units of central bank reserves. The cash market also allows for **central bank purchases of business loans** against reserves, denoted by $S(\bar{s}; R) \geq 0$, where $\bar{s}$ denotes the system-wide reserves in period 1. We assume that the central bank is never a good monitor of business loans. Like a bad bank, it therefore earns a gross return of 1, and no excess return, on its loan portfolio.[20]

Period 3: Loans pay off. One unit of business loans owned by a good bank pays $1 + R$, whereas one unit owned by a bad bank or by the central bank pays 1. One unit of interbank loans pays $1 + r$. The central bank redeems reserves using the proceeds from its stored goods and from the business loans it owns, and interbank loans are settled. We assume that bad banks can default on period-2 interbank loans because of the frictions in the credit market, as explained below, whereas good banks cannot.[21] Banks pay $1 + \gamma$ to their depositors. Any remaining resources constitute bank profits and are paid lump-sum to the households.[22] These profits may be negative. Households then consume.

[19] For a formal analysis, see Section B of the online appendix.

[20] Households, firms, bad banks, and the central bank do not have the capacity for good monitoring.

[21] The ability to default on period-2 interbank loans need not be tied to a lack of monitoring capacity; the results are unchanged if all banks, including good banks, may default. We make this assumption only to simplify the exposition and fix ideas, without loss of generality.

[22] Firms earn neither profits nor losses.

2.3 Financial Frictions and Equilibrium

Two financial frictions affect the functioning of the credit market.

1. **Asymmetric information.** We assume that idiosyncratic bank types are private information in period 2: whether a bank is good or bad is known only to the bank itself.

2. **Moral hazard.** Bad banks may default on their period-2 interbank loans and abscond with the proceeds of their business loans in period 3, while still repaying their depositors.[23] By contrast, good banks cannot default and always repay their interbank loans.

These financial frictions imply that bad banks may mimic good banks by purchasing business loans on credit and absconding with the proceeds in period 3 without repaying their interbank loans. As in Stiglitz and Weiss (1981) and Boissay et al. (2016, 2026), this possibility gives rise to **credit rationing**: a bank's interbank borrowing in the credit market, denoted by $\ell^{\text{cred}}(s, \tau; R, r, q)$, is constrained by

$$\ell^{\text{cred}}(s, \tau; R, r, q) \leq \bar{\ell}(s; r) \tag{4}$$

for some function $\bar{\ell}(s; r)$ of the credit-market rate r.[24]

A bank can also purchase up to s/q loans in the cash market. We impose that banks cannot short-sell loans. In sum, a bank chooses the quantity of loans traded in the period-2 credit market, $\ell^{\text{cred}} = \ell^{\text{cred}}(s, \tau; R, r, q)$, and in the period-2 cash market, $\ell^{\text{cash}} = \ell^{\text{cash}}(s, \tau; R, r, q)$, as functions of the reserves s acquired in period 1, its type τ, and market conditions (R, r, q), subject to the credit-rationing constraint (4) and to the following constraints:

$$\ell^{\text{cash}}(s, \tau; R, r, q) \quad \leq \quad \frac{s}{q} \tag{5}$$

$$\ell^{\text{total}}(s, \tau; R, r, q) \quad \geq \quad 0 \tag{6}$$

where

$$\ell^{\text{total}}(s, \tau; R, r, q) \equiv 1 - s + \ell^{\text{cred}}(s, \tau; R, r, q) + \ell^{\text{cash}}(s, \tau; R, r, q) \tag{7}$$

is the total end-of-period-2 loan portfolio: the loans $1 - s$ held initially plus the net quantities traded in the credit and cash markets. Equation (5) is a cash-in-advance

[23] Allowing banks to default on depositors would require introducing an additional financial friction between banks and households, giving bank leverage an explicit role. Since our focus is on bank liquidity rather than bank capital, we abstract from this complication and assume that there is no friction between banks and households.

[24] The borrowing limit $\bar{\ell}(s; r)$ is stated explicitly in equation (18) in the analysis section.

constraint, stating that the bank cannot buy more loans in the cash market than it can afford with its reserve holdings. The last constraint, (6), is a short-sale constraint on loans.

Ignoring payments to depositors, these choices imply the following period-3 payoff:

$$\begin{aligned}\Pi(s,\tau;R,r,q) &= s+(1+R)^{\tau=g}\ell^{\text{total}}(s,\tau;R,r,q)\\ &-(1+r)\ell^{\text{cred}}(s,\tau;R,r,q)-q\ell^{\text{cash}}(s,\tau;R,r,q)-\phi. \end{aligned} \tag{8}$$

This payoff is the sum of the payoffs on reserves and total loans held, net of credit liabilities, reserve payments in the cash market, and the screening cost. If loans are sold for credit or cash, the corresponding values of ℓ^{cred} or ℓ^{cash} are negative, so the terms above represent receipts rather than payments. We use the shorthand $(1+R)^{\tau=g}$ to denote a variable equal to $1+R$ if $\tau=g$, and equal to 1 if $\tau=b$.

Given Bertrand competition among banks and households' risk neutrality, the deposit return γ is such that expected profits, defined as expected payoffs net of deposit liabilities, are zero. We can now define an **equilibrium**.

Definition 1 (**Equilibrium**). *Given the liquidity regulation s_r, and the central bank's period-2 loan purchases $S(\overline{s};R)\geq 0$ as a function of the period-1 system-wide reserves $\overline{s}$ and business loan excess return R, an equilibrium consists of a period-1 reserve choice s and system-wide reserves $\overline{s}$, period-2 loan trading functions $\ell^{cred}(s,\tau;R,r,q)$ and $\ell^{cash}(s,\tau;R,r,q)$, a rate of return on deposits γ, a credit-market rate $r(R)$ as a function of R, a cash-market price of loans $q(R)$ as a function of R, and a credit limit $\overline{\ell}(s;r)$ as a function of s and r, such that:*

1. *Given s, bank type τ, and market conditions (R,r,q), the trading choices $\ell^{cred}(s,\tau;R,r,q)$ and $\ell^{cash}(s,\tau;R,r,q)$ maximize period-3 payoffs $\Pi(s,\tau;R,r,q)$ subject to the constraints (4), (5), and (6), together with the definition (7).*

2. *The credit limit $\overline{\ell}(s;r)$ is the highest limit consistent with bad banks not finding it profitable to purchase loans on credit in the credit market.*

3. *The symmetric reserve choice satisfies $s=\overline{s}$ and maximizes expected bank payoffs*

$$\overline{\Pi}(s)=E[\mu\Pi(s,b;R,r(R),q(R))+(1-\mu)\Pi(s,g;R,r(R),q(R))] \tag{9}$$

where the expectation is taken with respect to $R\sim F(R)$, subject to the liquidity regulation

$$s\geq s_r.$$

4. The deposit return γ is such that expected profits are zero,

$$1 + \gamma = \overline{\Pi}(s). \tag{10}$$

5. Given R, the credit-market rate $r = r(R)$ and the cash-market price of loans $q = q(R)$ clear the credit and cash markets:

$$0 = \mu\ell^{cred}(\overline{s}, b; R, r, q) + (1-\mu)\ell^{cred}(\overline{s}, g; R, r, q) \tag{11}$$

$$0 = \mu\ell^{cash}(\overline{s}, b; R, r, q) + (1-\mu)\ell^{cash}(\overline{s}, g; R, r, q) + S(\overline{s}; R). \tag{12}$$

In what follows, we write s rather than $\overline{s}$ whenever it is clear that s corresponds to the common reserve holdings chosen by all banks in period 1.

3 Analysis: Laissez-Faire

At the heart of the model lies the interbank credit market in period 2 and the financial frictions that affect it. Both the cash market and the credit market allow good banks to purchase business loans in period 2. Two scenarios may arise. In the best-case scenario, banks trade in the credit market, and business loans are sold to good banks. In the worst-case scenario, asymmetric information and moral hazard prevent credit from flowing to good banks: bad banks may mimic good banks as borrowers, purchase loans on credit, and default. In this case, the credit market freezes, and banks can trade only in the cash market.

The greater the amount of reserves banks hold, the larger the volume of trade that can take place in the cash market and the less severe the detrimental effect of a credit-market freeze. Central bank reserves therefore allow the cash market to act as a liquidity buffer, immune to information frictions, in states of nature in which the credit market malfunctions. However, holding central bank reserves crowds out business loans and profitable investment in period 1. Banks' optimization problem thus involves weighing the opportunity cost of liquidity insurance against its benefits in times of stress.

In this section, we analyze the laissez-faire situation, where there is neither liquidity regulation, $s_r = 0$, nor central bank purchases, $S(\overline{s}; R) \equiv 0$. We analyze the cash market first. The remaining unsold loans are then available for trade on the credit market. This sequential approach to the analysis is convenient but is not meant to imply a sequence in which these two markets clear; in fact, they clear simultaneously.

3.1 Trades on the Cash Market

Construction of the Supply Schedule $L_S^{\mathbf{cash}}(s; q)$. The μ bad banks are the natural sellers of business loans on the cash market. Given s, R, and r, write the total loan supply on the cash market as

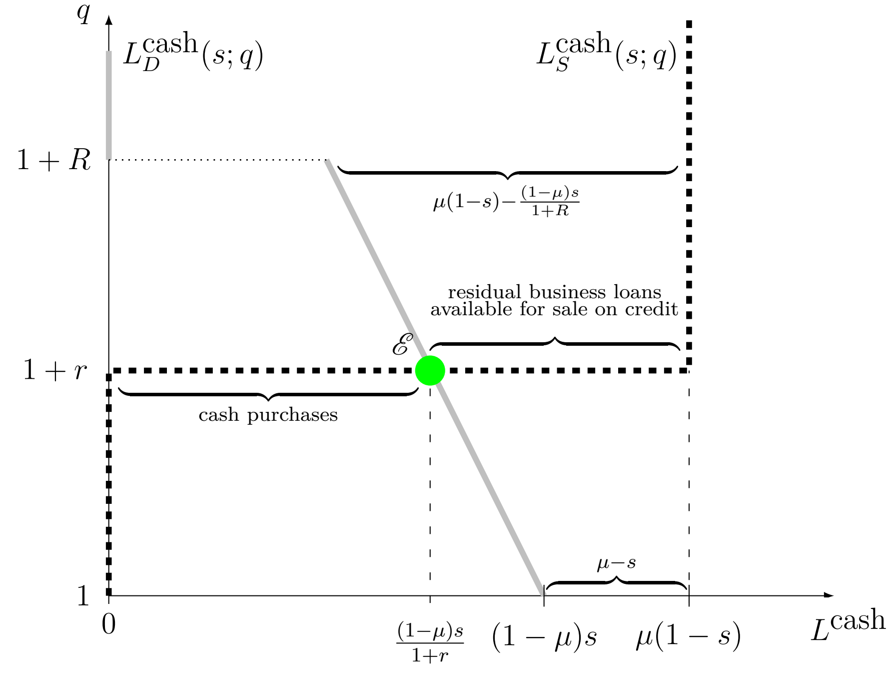
$$L_S^{\text{cash}}(s;q) = -\mu \ell^{\text{cash}}(s, b; R, r, q).$$

Bad banks may sell their business loans either to good banks for q units of central bank reserves or on the credit market against a promise to receive $1+r$ units of the consumption good in period 3. A bad bank sells its business loan portfolio $1-s$ on the cash market if $q > 1+r$; in that case, the supply of business loans is $L_S^{\text{cash}}(s;q) = \mu(1-s)$. The bad bank does not sell if $q < 1+r$; in that case, $L_S^{\text{cash}}(s;q) = 0$. For positive trades on both markets, it therefore must be the case that[25]

$$q = 1 + r. \tag{13}$$

In that case, $L_S^{\text{cash}}(s;q) \in [0, \mu(1-s)]$. This yields the dashed supply curve $L_S^{\text{cash}}(s;q)$ in Figure 2.

Figure 2: Cash Market in Period 2



Notes: Plot of the business loan supply (dashed line) and demand (gray line) schedules on the secondary loan-for-reserves market, for the case with $s < \mu$.

Construction of the Demand Schedule $L_D^{\text{cash}}(s;q)$. The $1-\mu$ good banks are the natural buyers of business loans in period 2. Given s, R, and r, the total loan demand on the cash market is

$$L_D^{\text{cash}}(s;q) = (1-\mu)\ell^{\text{cash}}(s, g; R, r, q).$$

[25] If there is no trade on one of the markets, we impose (13) without loss of generality.

Good banks may buy business loans on the cash market with central bank reserves.[26] At the price q, their reserve holdings s allow each good bank to purchase up to s/q business loans. These purchased loans pay $1+R$ in period 3. Hence, good banks purchase as many business loans as possible if $q < 1+R$. When $q = 1+R$, good banks are indifferent. In that case, we assume that they still spend all their reserves to purchase business loans. When $q > 1+R$, good banks do not purchase any loans. Thus, $L_D^{\text{cash}}(s;q) = (1-\mu)s/q$ if $q \leq 1+R$ and $L_D^{\text{cash}}(s;q) = 0$ if $q > 1+R$. This yields the demand curve $L_D^{\text{cash}}(s;q)$ in Figure 2. In the competitive equilibrium of the cash market, demand equals supply,

$$L_D^{\text{cash}}(s;q) = L_S^{\text{cash}}(s;q), \tag{14}$$

which determines the volume of loans traded on the cash market.

Residual Amount of Business Loans Available for Sale on the Credit Market. Given the equilibrium conditions (13) and (14), the residual supply of business loans available for sale on credit is given by

$$L_S^{\text{cred}}(s;r) \equiv -\mu\ell^{\text{cred}}(s,b;R,r,q) \begin{cases} = \mu(1-s) & \text{if } r > R \\ = \max\left\{0 \;;\; \mu(1-s) - \frac{(1-\mu)s}{1+r}\right\} & \text{if } r \in (0,R] \\ \in [0 \;,\; \max\{0 \;;\; \mu - s\}] & \text{if } r = 0. \end{cases} \tag{15}$$

When $r > R$, $q > 1+R$, and no good bank buys business loans for cash. In that case, the residual supply of business loans on the credit market corresponds to bad banks' entire portfolio, $\mu(1-s)$. When $r \in (0,R]$, good banks purchase $1/q = 1/(1+r)$ business loans per unit of reserves, and the residual supply of business loans available for sale on credit is equal to $\mu(1-s) - (1-\mu)s/(1+r)$ if this amount is positive and to zero otherwise. When $r = 0$, bad banks are indifferent between selling their business loans on the cash market, selling them on the credit market, and keeping them on their balance sheets. In that case, they may supply up to $\mu - s$ loans to the market if this amount is positive, and nothing otherwise. This yields the aggregate loan supply curve $L_S^{\text{cred}}(s;r)$ in Figure 3. The residual supply of business loans available for sale on the credit market is strictly positive for all feasible credit-market rates if and only if

$$\mu(1-s) > \frac{(1-\mu)s}{1+r} \quad \forall r \in [0,R],$$

or, equivalently, if and only if

$$s < \mu, \tag{16}$$

[26] The alternative for a good bank would be to keep its reserves on its balance sheet and obtain more loans on the credit market. The equilibrium outcome would be unchanged, as shown in the online appendix, Section C.

which corresponds to a case where good banks do not hold enough central bank reserves in period 1 to purchase all bad banks' business loans for cash in period 2. As we shall show, this condition is satisfied in the laissez-faire equilibrium as well as under any of the policies considered: liquidity regulation, central bank loan purchases, and the policy mix.

3.2 Frictionless Credit Market

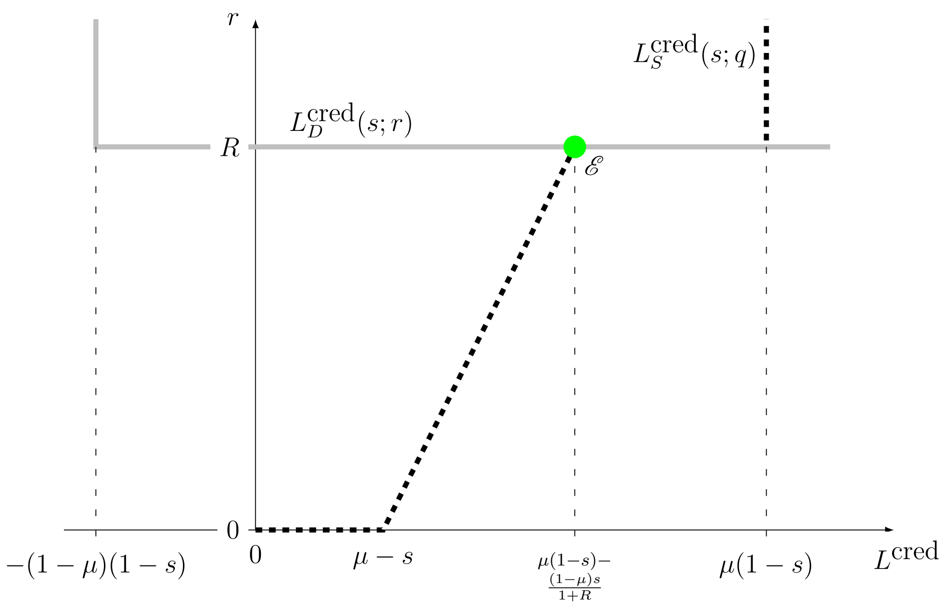
As a benchmark, we first study the credit market in the absence of the financial frictions described in subsection 2.3. An interbank loan contract consists of the volume of business loans $\ell^{\text{cred}}(s, \tau; R, r, q)$ purchased on credit by an individual borrower and the interest rate r.

The aggregate amount of business loans available for sale on the credit market, $L_S^{\text{cred}}(s; r)$, is given by (15).

The $1-\mu$ good banks are the natural buyers of business loans, implying $L_D^{\text{cred}}(s; r) = (1-\mu)\ell^{\text{cred}}(s, g; R, r, q)$. If $r < R$, they buy as many business loans as possible, implying $L_D^{\text{cred}}(s; r) = +\infty$. If $r = R$, good banks are indifferent between buying and selling loans: $L_D^{\text{cred}}(s; r) \in [-(1-\mu)(1-s), +\infty)$. If $r > R$, they sell their entire portfolio of $1-s$ business loans: $L_D^{\text{cred}}(s; r) = -(1-\mu)(1-s)$. The aggregate credit demand by good banks is therefore given by (17) and represented by the gray curve in Figure 3.

$$L_D^{\text{cred}}(s; r) \equiv (1-\mu)\ell^{\text{cred}}(s, g; R, r, q) \begin{cases} = -(1-\mu)(1-s) & \text{if } r > R \\ \in [-(1-\mu)(1-s), +\infty) & \text{if } r = R \\ = +\infty & \text{if } r < R. \end{cases} \tag{17}$$

Figure 3: Frictionless Credit Market



The credit market clears when

$$L_D^{\text{cred}}(s;r) = L_S^{\text{cred}}(s;r),$$

which yields the unique equilibrium solution $r = R$ and therefore $q = 1+R$. In equilibrium, good banks buy all residual business loans available for sale on the credit market, for any realization of R, and business loans are fully and efficiently reallocated from bad to good banks in period 2. Theorem 1 follows.

Theorem 1 (**First Best Outcome**). *Absent financial frictions, the credit market channels all residual business loans to good banks, while $r = R$ and $q = 1 + R$.*

3.3 Frictional Credit Market

We now consider the case in which the credit market is subject to the two financial frictions, asymmetric information and moral hazard, described in subsection 2.3. .

Construction of the Demand Schedule $L_D^{\text{cred}}(s;r)$. With the two financial frictions, a bad bank may mimic a good bank, purchase loans on credit, and default. We assume that its balance sheet is observable by lenders.[27] A mimicking bad bank therefore purchases s/q business loans on the cash market, as a good bank would, and $\ell^{\text{cred}}(s, g; R, r, q)$ business loans on the credit market, like a good bank. It ends up with $1-s+s/q+\ell^{\text{cred}}(s, g; R, r, q)$ business loans on its balance sheet. Since these loans return one consumption good per business loan, the mimicking bad bank receives $1-s+s/q+\ell^{\text{cred}}(s, g; R, r, q)$ consumption goods as gross return on its loan portfolio in period 3, while defaulting on its credit obligations.

By contrast, a bad bank that does not mimic good banks sells its $1 - s$ business loans at rate r on the credit market (or at price $q = 1 + r$ on the cash market) and thus earns $(1 - s)(1 + r)$ plus the return on its reserves at the central bank, s, in period 3.

To prevent bad banks from mimicking good banks, it must be incentive-compatible for bad banks not to do so.[28] This gives the incentive-compatible borrowing constraint

$$\underbrace{1 - s + \frac{s}{q} + \ell^{\text{cred}}(s, g; R, r, q)}_{\substack{\text{mimics a good bank,}\\ \text{purchases business loans on credit}\\ \text{and defaults}}} \leq \underbrace{(1 - s)(1 + r) + s}_{\text{sells business loans on credit}} \tag{IC}$$

[27]Otherwise, the bank could split its borrowing across many lenders. Such a credit market would collapse entirely.

[28]An implicit assumption here is that defaults are so costly to lenders that no lender is willing to tolerate them. For an analysis of the possibility of a pooling equilibrium that includes defaults, refer to the online appendix in Boissay et al. (2026).

Using (13), *i.e.*, $q = 1 + r$, this yields the credit-rationing constraint

$$\ell^{\text{cred}}(s, g; R, r, q) \leq \bar{\ell}(s; r) \equiv r\left[1 - \frac{r}{1+r}s\right], \tag{18}$$

where $\bar{\ell}(s; r)$ *increases* with r. If $r > R$, good banks do not buy business loans on credit because doing so would generate losses; instead, they prefer to sell their loans on credit: $L_D^{\text{cred}}(s; r) = -(1 - \mu)(1 - s)$. If $r < R$, the $1 - \mu$ good banks purchase the maximum amount of business loans on credit subject to the credit-rationing constraint, and their borrowing limit binds, so that

$$\begin{aligned} L_D^{\text{cred}}(s; r) =& (1 - \mu)\bar{\ell}(s; r) \\ =& (1 - \mu)r\left[1 - \frac{r}{1+r}s\right], \end{aligned} \tag{19}$$

which also *increases* with r. If $r = R$, good banks are indifferent about how many business loans they buy or sell on credit: $L_D^{\text{cred}}(s; r) \in [-(1 - \mu)(1 - s), (1 - \mu)\bar{\ell}(s; r)]$. This yields

$$L_D^{\text{cred}}(s; r) \equiv (1 - \mu)\ell^{\text{cred}}(s, g; R, r, q) \begin{cases} = -(1 - \mu)(1 - s) & \text{if } r > R \\ \in \left[-(1 - \mu)(1 - s), (1 - \mu)R\left[1 - \frac{R}{1+R}s\right]\right) & \text{if } r = R \\ = (1 - \mu)r\left[1 - \frac{r}{1+r}s\right] & \text{if } r < R. \end{cases} \tag{20}$$

as represented by the gray demand curves in Figure 4.

In sum, as in Stiglitz and Weiss (1981) and Boissay et al. (2016, 2026), lenders ration the amount of credit that borrowers can obtain in order to prevent default. All banks face the same borrowing limit (18), which ensures that bad banks have no incentive to mimic good banks by borrowing to purchase loans and then defaulting. Equivalently, the constraint makes a bad bank better off selling its business loan portfolio on credit at rate r than purchasing business loans on credit.

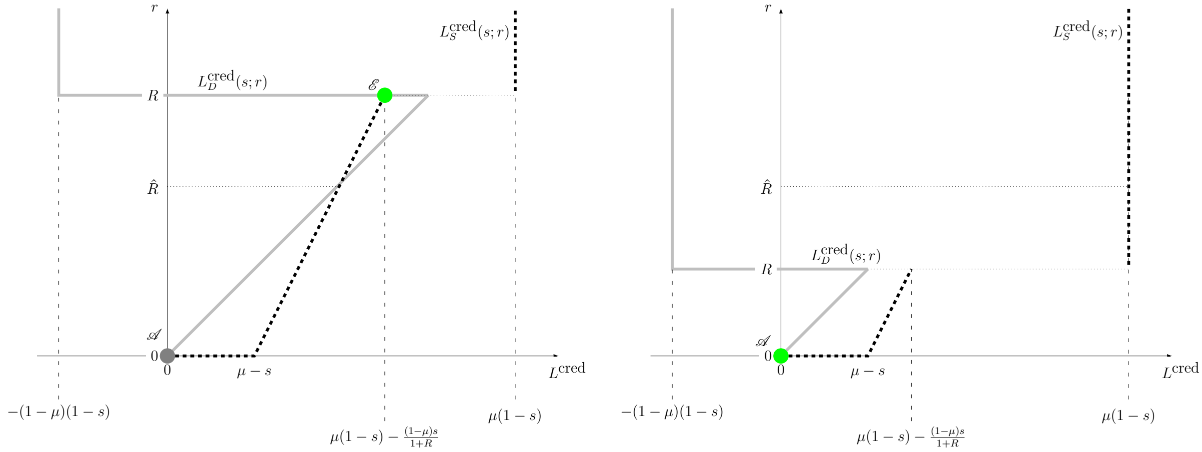
Supply Schedule $L_S^{\text{cred}}(s; r)$. Financial frictions do not alter the functioning of the cash market itself: conditional on (s, R, r, q), the cash-market supply and demand schedules are the same as in the frictionless case.[29] Given this, the residual supply of loans on the credit market is still given by expression (15) and represented by the dashed curve $L_S^{\text{cred}}(s; r)$ in Figure 4. The credit market clears when

$$L_D^{\text{cred}}(s; r) = L_S^{\text{cred}}(s; r).$$

[29] They may, however, affect the cash-market equilibrium through their effect on the credit-market rate r and, therefore, on the cash-market price $q = 1 + r$.

Figure 4: Frictional Credit Market

(i) Case with $R \geq \hat{R}$ (ii) Case with $R < \hat{R}$



Abstracting from coordination failures that could lead to an inefficient no-trade equilibrium, we select the efficient equilibrium and denote it by $\mathscr{E}$.[30]

Theorem 2 (**Market Freeze**). *In the presence of financial frictions, the credit market freezes if the realized excess return on loans R is low. Specifically, no trade occurs when $R < \hat{R}(s)$, where the crisis threshold $\hat{R}(s)$ is defined as*

$$\hat{R}(s) \equiv \max\left\{0 \ ; \ \frac{\mu - s}{(1-\mu)(1-s)}\right\}. \tag{21}$$

Furthermore, the probability of a market freeze, $F(\hat{R})$, decreases with the aggregate level of reserves s and increases with the mass μ of bad banks, and is fully eliminated if $s \geq \mu$.

Proof. The market freezes when supply exceeds incentive-compatible demand for all feasible interest rates $r \in (0, R]$:

$$L_D^{\text{cred}}(s;r) < L_S^{\text{cred}}(s;r) \ \ \forall r \in (0,R], \ \forall s \in (0,\mu).$$

Using (15) and (19), the above condition is equivalent to

$$(1-\mu)r\left[1 - \frac{r}{1+r}s\right] < \mu(1-s) - \frac{(1-\mu)s}{1+r} \ \ \forall r \in (0,R], \ \forall s \in (0,\mu),$$

[30]When $R \geq \hat{R}$ (Figure 4i), equilibria $\mathscr{E}$ and $\mathscr{A}$ coexist, and banks may coordinate on either one. In what follows, we rule out coordination failures and assume that banks always coordinate on $\mathscr{E}$, the efficient equilibrium. There are several ways to justify this assumption. First, a mere commitment by the central bank to purchase assets at the equilibrium cash price prevailing under $\mathscr{E}$ (*i.e.* $q = 1 + R$) is sufficient to coordinate banks on $\mathscr{E}$, without requiring the central bank to purchase any assets in equilibrium. Section D in the online appendix provides a complementary justification. It shows that, unlike $\mathscr{E}$, equilibrium $\mathscr{A}$ is not robust to individual price deviations in all states of nature when $R \geq \hat{R}$. By contrast, $\mathscr{A}$ is robust to such deviations when $R < \hat{R}$ (Figure 4ii).

or

$$r < \frac{\mu - s}{(1-\mu)(1-s)} \quad \forall r \in (0, R],\ \forall s \in (0, \mu).$$

Thus, a freeze occurs when $R < \hat{R}(s)$, where

$$\hat{R}(s) = \max\left\{0\ ;\ \frac{\mu - s}{(1-\mu)(1-s)}\right\}.$$

For $s < \mu$, differentiating the interior threshold with respect to s and μ yields

$$\frac{\partial \hat{R}}{\partial s} = \frac{-1}{(1-s)^2} < 0 \qquad \text{and} \qquad \frac{\partial \hat{R}}{\partial \mu} = \frac{1}{(1-\mu)^2} > 0.$$

If $s \geq \mu$, then $\hat{R}(s) = 0$, so the market-freeze probability is zero. □

Theorem 2 shows that, for the credit market to operate, the excess return on good banks' business loans, R, must be sufficiently high for good banks to offer a return that deters bad banks from borrowing and absconding.

When $R \geq \hat{R}$, all business loans are reallocated to good banks, and financial frictions have no material effect on the interbank market. We refer to this case as "normal times". In normal times, the market for business loans is tight: strong demand keeps prices high and allows sellers—namely, bad banks—to capture the surplus. In equilibrium, $r = R$ and $q = 1 + R$, so the allocation coincides with the frictionless benchmark, as illustrated by equilibrium $\mathscr{E}$ in Figure 4i.

By contrast, when $R < \hat{R}$, the credit market freezes because potential lenders cannot distinguish good borrowers from bad borrowers who may default. In that case, the tightness of the market for business loans reverses. The borrowing constraint (IC) binds, and good banks become financially constrained. Their demand for credit weakens, the equilibrium credit-market rate falls to $r = 0$, and no trade takes place, as illustrated by equilibrium $\mathscr{A}$ in Figure 4ii. We refer to this case as a "crisis".

In a crisis, bad banks keep on their balance sheets the business loans that they cannot sell in the cash market. In that market, the price of business loans falls to $q = 1$ (from (13)), below the normal-times price, $q = 1 + R$. This lower price allows good banks to purchase more business loans with their reserves than they could in normal times. Central bank reserves are therefore especially valuable in a crisis: their purchasing power rises precisely when liquidity is scarce, mitigating the cost of the credit-market freeze.

As long as $s < \mu$, however, banks' reserve holdings are not sufficient to achieve full reallocation through the cash market. When the credit market freezes, only a volume $(1-\mu)s$ of business loans is reallocated to good banks through the cash market. Since bad banks initially hold a volume $\mu(1-s)$ of business loans, a volume $\mu(1-s) - (1-\mu)s = \mu - s > 0$ remains on bad banks' balance sheets and yields no excess return. Theorem 2 further shows that the probability of a credit-market freeze increases with the share of bad banks, μ, and decreases with the stock of reserves, s, available to good banks for purchasing business loans. A lower μ reduces the mass of bad banks that must be deterred from borrowing, thereby making a freeze less likely. Similarly, a higher s reduces the residual supply of business loans that must be absorbed in the credit market, allowing the market to clear even when R is low and incentive-compatible demand is constrained.

In what follows, it is useful to define the function $s(\hat{R})$ as

$$s(\hat{R}) \equiv \max\left\{0, \frac{\mu - (1-\mu)\hat{R}}{1-(1-\mu)\hat{R}}\right\}. \tag{22}$$

This function denotes the level of banks' period-1 reserve holdings that generates the crisis threshold $\hat{R}$.

3.4 Payoffs in Period 3

Payoffs in Normal Times ($R \geq \hat{R}$). In normal times, the $\mu(1-s)$ business loans originated by the bad banks are reallocated efficiently to the $1-\mu$ good banks and yield excess return R. The equilibrium interbank loan rate settles at $r(R) = R$, and all banks ultimately obtain the excess return R on their business loans. There is no excess return on reserves. With

$$\Pi^n(s; R) = \mu\Pi(s, b; R, R, 1+R) + (1-\mu)\Pi(s, g; R, R, 1+R)$$

as the total bank payoff in normal times (see equation (8)), gross of payments to depositors, we obtain

$$\Pi^n(s; R) = (1-s)(1+R-\kappa) + s. \tag{23}$$

Payoffs in a Crisis ($R < \hat{R}$). In a crisis, the reallocation of business loans to good banks occurs only through the cash market and is partial. Each good bank purchases s additional business loans from bad banks with its s central bank reserves at a price of $q = 1$, implying $r = 0$ per (13).[31] These loans add to the bank's own $1-s$ business loans. Hence, a good bank obtains excess return $(1-s+s)R$. The mass μ of bad banks sell some of their business loans to good banks on the cash market against reserves and keep the rest on their balance sheets. Bad banks also hold their initial reserves s. Since neither reserves nor business loans yield any excess return for bad banks, their excess return is zero. With

$$\Pi^c(s; R) = \mu\Pi(s, b; R, 0, 1) + (1-\mu)\Pi(s, g; R, 0, 1)$$

[31] There are no transactions on the credit market, so r is irrelevant.

as the total bank payoff in crisis states, we obtain

$$\begin{aligned}\Pi^c(s;R) =& \mu(1-s+s)+(1-\mu)(1-s+s)(1+R)-\kappa(1-s)\\ =&(1-s)(1+R-\kappa)+s-(\mu-s)R. \end{aligned} \tag{24}$$

Compared to $\Pi^n(s;R)$ in (23), the resources $(\mu-s)R$ are lost, reflecting the business loans $\mu-s$ beyond the cash market that cannot be traded on the credit market.

3.5 Welfare Function and Return-at-Risk

Welfare is measured by households' expected consumption in period 3. In the laissez-faire economy, it is equal to expected bank payoffs, as defined in equation (9). Let $W(s;\hat{R})$ denote welfare as a function of banks' reserve holdings s and the crisis threshold $\hat{R}$. Given the calculations above,[32]

$$W(s;\hat{R}) = (1-s)(1+\rho-\kappa)+s-(\mu-s)\Psi(\hat{R}), \tag{25}$$

where

$$\Psi(\hat{R}) \equiv \int_0^{\hat{R}} Rf(R)dR \tag{26}$$

is the expected return forgone, per unit of misallocated loans, when business loans remain on bad banks' balance sheets in crisis states. In the analysis that follows, it will be convenient to use the following notation interchangeably:

$$W(s) = W(s;\hat{R}(s)) \tag{27}$$

and

$$W(\hat{R}) = W(s(\hat{R});\hat{R}), \tag{28}$$

where $\hat{R}(s)$ and $s(\hat{R})$ are given by (21) and (22), respectively. It is also useful to introduce the notion of *return-at-risk*.

Definition 2 (Return-at-Risk). *The return-at-risk is defined as*

$$\text{RaR} \equiv \Psi\left(\frac{\mu}{1-\mu}\right). \tag{29}$$

The return-at-risk, RaR, measures the expected return forgone in crisis states per unit of business loans left with bad banks in period 2, evaluated in the laissez-faire economy with zero reserve holdings, $s=0$. In this case, the crisis threshold is $\hat{R}=\mu/(1-\mu)$, the crisis probability is $F\left(\mu/(1-\mu)\right)$, and the expected return forgone per unit of misallocated loans is $\Psi\left(\mu/(1-\mu)\right)$. This corresponds to the worst-case left-tail scenario: expected crisis

[32]Relation (25) follows from $W(s,\hat{R}) = \int_0^{\hat{R}} \Pi^c(s;R)f(R)dR + \int_{\hat{R}}^{+\infty} \Pi^n(s;R)f(R)dR$.

losses are maximal because no reserves are available to reallocate business loans from bad banks to good banks. Since $\Psi\left(\mu/(1-\mu)\right) < \Psi(+\infty)$ and $\rho = \int_0^{+\infty} Rf(R)dR = \Psi(+\infty)$, it follows that RaR $< \rho$. RaR is useful because it captures crisis risk independently of any policy intervention and can therefore be interpreted as a primitive, policy-invariant measure of left-tail risk.

3.6 Demand for Reserves and Welfare Outcome

Banks maximize expected profit $\overline{\Pi}(s)$ or, equivalently, welfare $W(s;\hat{R})$, with respect to their reserve holdings s, taking the aggregate crisis threshold $\hat{R}$ as given. Individual banks do not internalize the effect of system-wide reserve holdings $\overline{s}$ on $\hat{R}(\overline{s})$. We obtain the first-order condition

$$\Psi(\hat{R}^{\star}) = \rho - \kappa. \tag{30}$$

The laissez-faire crisis threshold $\hat{R}^{\star}$ can be obtained from (30) by inverting Ψ. Since $f(R)$ is strictly positive on $\mathbb{R}^{+}$ and $\Psi'(R) = Rf(R)$, there is a unique solution for $\hat{R}^{\star}$.

Relation (30) captures a bank's risk–return trade-off. On the one hand, holding reserves is costly because it crowds out investment in profitable business loans, whose expected net excess return is $\rho - \kappa$. On the other hand, if the bank turns out to be good in period 2, reserves allow it to purchase profitable business loans from bad banks during a crisis at a low cash-in-the-market price. This liquidity-insurance benefit is captured by $\Psi(\hat{R}^{\star})$. In that sense, reserves provide insurance within the banking system by ensuring that banks can trade and reallocate at least some business loans in a crisis. The larger the aggregate stock of reserves held by banks, the lower the crisis threshold $\hat{R}(\overline{s})$ in (21) and, hence, the lower the probability of a crisis. This, in turn, lowers the private marginal benefit of holding reserves. Since $\kappa < \rho$, relation (30) implies that $\hat{R}^{\star} > 0$, which means that banks do not find it optimal to hold enough reserves to eliminate credit-market freezes altogether. Crises therefore occur with positive probability. Given (21), this implies that $s^{\star} < \mu$.

An equilibrium with $s^{\star} = 0$ is compatible with the analysis above. In what follows, we focus on the more interesting case in which the laissez-faire equilibrium is interior, with $s^{\star} > 0$. To do so, we restrict the set of parameters of the model so that

$$\hat{R}^{\star} < \hat{R}(0) = \frac{\mu}{1-\mu}. \tag{31}$$

Thus, we make the following assumption, which is equivalent to (31), given (30):

Assumption 1. RaR $\equiv \Psi\left(\frac{\mu}{1-\mu}\right) > \rho - \kappa$.

Assumption 1 formalizes the trade-off that determines whether banks hold a strictly positive amount of reserves in period 1. Recalling Definition 2, RaR is the expected return forgone in crises per unit of misallocated business loans when $s = 0$. In a crisis, one unit of reserves allows good banks to purchase one unit of business loans from bad banks, since $q = 1$ in that state. Thus, starting from $s = 0$, one additional unit of reserves reduces the expected cost of a crisis by RaR. At the same time, it crowds out investment in business loans, lowering banks' net expected return by $\rho - \kappa$. Assumption 1 requires the former effect to dominate the latter: RaR $> \rho - \kappa$. Hence, in the laissez-faire economy, banks optimally choose a strictly positive amount of reserves.

The opposite margin pins down the relevant upper bound. If $s \geq \mu$, reserves are sufficiently abundant to absorb all business loans held by bad banks in a crisis. The crisis threshold is then $\hat{R} = 0$, so the expected crisis cost is zero, $\Psi(0) = 0$. At that point, an additional unit of reserves yields no marginal liquidity benefit, while still imposing the opportunity cost $\rho - \kappa > 0$. Banks therefore never choose reserves large enough to eliminate credit-market freezes altogether. The laissez-faire allocation consequently satisfies $0 < s^\star < \mu$.

Optimal Demand for Reserves under Laissez-Faire. Given Assumption 1 and relation (30), the level of laissez-faire reserves $s^\star$ can be obtained from equation (22) as

$$s^\star = \frac{\mu - (1-\mu)\Psi^{-1}(\rho - \kappa)}{1 - (1-\mu)\Psi^{-1}(\rho - \kappa)} \in (0, \mu). \tag{32}$$

Banks' optimal demand for reserves decreases with the expected return on business loans net of origination and screening costs, $\rho - \kappa$, and increases with the mass of bad banks, μ. Intuitively, a larger mass of bad banks raises the likelihood of a crisis in period 2 and therefore increases the value of precautionary reserves.

Welfare Outcome under Laissez-Faire. Laissez-faire welfare can then be computed by using the first-order condition (30) to substitute $\rho - \kappa$ for $\Psi(\hat{R}(s))$ in equation (25), which yields

$$W^\star = 1 + (1-\mu)(\rho - \kappa). \tag{33}$$

4 Policy Interventions

To improve on the laissez-faire allocation, we consider two forms of central-bank intervention: preventive liquidity regulation, imposed *ex ante* in period 1, and asset purchases conducted *ex post* in period 2, through which the central bank acts as BOLR.

4.1 Liquidity Regulation ($s \geq s_r$)

Liquidity regulation requires banks to hold at least s_r units of central bank reserves in period 1. In what follows, the subscript r denotes variables evaluated under the socially optimal liquidity requirement.

Optimal Minimum Liquidity Requirement s_r. Unlike individual banks, the regulator internalizes the effect of aggregate reserve holdings on the probability and cost of market freezes. The regulator therefore chooses the liquidity requirement s_r to maximize welfare in (25), subject to the relationship between the reserve buffer s_r and the crisis threshold $\hat{R}_r$ given by (21). This relationship allows us to express the regulator's problem directly in terms of the crisis threshold. Rearranging terms in (25) and using (21) to substitute out s_r, welfare can be written as

$$W(\hat{R}_r) = \; 1 + (1-\mu)(\rho - \kappa) + \frac{(1-\mu)^2 \hat{R}_r}{1-(1-\mu)\hat{R}_r}\left[\rho - \kappa - \Psi(\hat{R}_r)\right]. \tag{34}$$

Taking the first-order condition of (34) with respect to the crisis threshold $\hat{R}_r$, using $\Psi'(R) = Rf(R)$, and multiplying through by $\left[1-(1-\mu)\hat{R}_r\right]^2$, yields

$$\Psi(\hat{R}_r) + \underbrace{\left[1-(1-\mu)\hat{R}_r\right]\hat{R}_r^2 f(\hat{R}_r)}_{\text{pecuniary externality}} = \rho - \kappa. \tag{35}$$

The second term on the left-hand side is strictly positive at any interior threshold. It captures the marginal effect of aggregate reserve holdings on the incidence and cost of financial crises: higher reserves allow more business loans to be transferred from bad banks to good banks through the cash market, thereby reducing the scale of business-loan misallocation in a crisis. Since individual banks do not internalize this effect, it constitutes a pecuniary externality. Define

$$\Psi_r(\hat{R}) \equiv \Psi(\hat{R}) + \left[1-(1-\mu)\hat{R}\right]\hat{R}^2 f(\hat{R}). \tag{36}$$

The regulatory first-order condition (35) can then be written compactly as

$$\Psi_r(\hat{R}_r) = \rho - \kappa. \tag{37}$$

This condition parallels the private first-order condition (30), except that the regulator equates the opportunity cost of reserves to $\Psi_r(\hat{R}_r)$, rather than to $\Psi(\hat{R}_r)$. Moreover, for any interior threshold,

$$\Psi_r(\hat{R}) > \Psi(\hat{R}). \tag{38}$$

The inequality reflects the pecuniary externality associated with aggregate reserve holdings. By increasing reserves, banks expand the volume of business loans that can be reallocated from bad banks to good banks in the cash market, thereby reducing business-loan misallocation in a crisis. The wedge $\Psi_r(\hat{R}) - \Psi(\hat{R})$ captures this additional marginal benefit of reserves.

For (37) to admit a unique solution, it is sufficient that $\Psi_r(\hat{R})$ be strictly increasing over the relevant range of crisis thresholds. We impose the following sufficient condition.[33]

Assumption 2. *The distribution of the aggregate shock, $f(R)$, satisfies*

$$\frac{\hat{R}f'(\hat{R})}{f(\hat{R})} > -3$$

for all feasible values of the crisis threshold, that is, for all $\hat{R} \leq \mu/(1-\mu)$.

Lemma 1. *Under Assumption 2, $\Psi_r(\hat{R})$ is strictly increasing on $[0, \mu/(1-\mu)]$. If Assumption 1 also holds, then the first-order condition (37) admits a unique solution $\hat{R}_r$.*

Proof. Differentiating (36) and using $\Psi'(R) = Rf(R)$, we obtain

$$\Psi_r'(\hat{R}) = \hat{R}f(\hat{R})\left[1-(1-\mu)\hat{R}\right]\left[3+\frac{\hat{R}f'(\hat{R})}{f(\hat{R})}\right].$$

Under Assumption 2, this derivative is strictly positive on $[0, \mu/(1-\mu)]$, which is the relevant range for the values of $\hat{R}$. As a result, the first-order condition (37) admits a unique solution. □

Substituting the solution $\hat{R}_r$ of (37) into relation (22) and rearranging yields the socially optimal reserve requirement. This expression is analogous to the private reserve demand in (32), except that it involves $\Psi_r^{-1}(\rho-\kappa)$ rather than $\Psi^{-1}(\rho-\kappa)$. The result is summarized in the following theorem.

Theorem 3 (Optimal Liquidity Regulation). *The optimal reserve requirement is*

$$s_r = \frac{\mu-(1-\mu)\Psi_r^{-1}(\rho-\kappa)}{1-(1-\mu)\Psi_r^{-1}(\rho-\kappa)} \in (s^\star, \mu). \tag{39}$$

This liquidity requirement exceeds banks' privately optimal reserve holdings and, therefore, binds.

The strict inequality $s_r > s^\star$ follows from the pecuniary externality. Since $\Psi_r(\hat{R}) > \Psi(\hat{R})$ at any interior threshold, the solution to the regulatory first-order condition satisfies $\hat{R}_r < \hat{R}^\star$. Because the reserve buffer is decreasing in the crisis threshold, this implies $s_r > s^\star$. Moreover, the regulator never sets $s_r = \mu$. As $s_r \nearrow \mu$, good banks have enough

[33]This condition is satisfied by standard distributions on $\mathbb{R}^+$, such as the exponential, Gamma, log-normal, and uniform distributions, for the relevant values of $\hat{R} \leq \mu/(1-\mu)$ and suitable values of $\mu \in (0,1)$.

reserves to purchase the entire loan portfolio of bad banks in period 2, so business-loan misallocation vanishes. At that point, there is no benefit from further increasing reserves, whereas the cost in terms of crowding out profitable business loans remains positive. Hence, the optimum satisfies $s_r < \mu$.

Unlike $\Psi(\hat{R})$ in (26), the regulatory marginal-benefit term $\Psi_r(\hat{R})$ in (36) depends on μ. Holding $\hat{R}$ fixed, a larger mass of bad banks raises $\Psi_r(\hat{R})$, since

$$\frac{\partial \Psi_r(\hat{R})}{\partial \mu} = \hat{R}^3 f(\hat{R}) > 0.$$

Under Assumption 2, $\Psi_r(\hat{R})$ is also strictly increasing in $\hat{R}$. Therefore, the regulatory first-order condition (37) implies that $\hat{R}_r$ decreases with μ. Intuitively, for a given liquidity requirement s_r, the extent of business-loan misallocation during a credit-market freeze, $\mu - s_r$, increases with μ, and the pecuniary externality becomes stronger. The regulator responds by raising the liquidity requirement s_r, thereby lowering the crisis threshold and reducing the probability and severity of a market freeze.

Welfare Outcome Under the Optimal Liquidity Requirement. Using the regulatory first-order condition (37), welfare at the regulatory optimum can be written as

$$W_r = \underbrace{1 + (1-\mu)(\rho - \kappa)}_{W^\star} + \chi_r, \tag{40}$$

where

$$\chi_r \equiv (\mu - s_r)\left[\rho - \kappa - \Psi(\hat{R}_r)\right]. \tag{41}$$

Here, $\hat{R}_r$ is determined by (37), and s_r is given by (39). Since $s_r < \mu$, and since (36)-(37) implies

$$\rho - \kappa - \Psi(\hat{R}_r) = \left[1 - (1-\mu)\hat{R}_r\right] \hat{R}_r^2 f(\hat{R}_r) > 0,$$

we have $\chi_r > 0$. Hence, welfare is strictly higher under the socially optimal liquidity requirement than under laissez-faire.

Theorem 4 (**Welfare Comparison**). *Optimal liquidity regulation strictly improves welfare relative to laissez-faire:*

$$W_r > W^\star.$$

Moreover, the optimal liquidity requirement induces higher reserve holdings and a lower crisis probability:

$$s_r > s^\star \qquad \text{and} \qquad F(\hat{R}_r) < F(\hat{R}^\star).$$

4.2 Central Bank Market Backstop

The second policy instrument available to the central bank is a backstop policy: in period 2, the central bank purchases business loans in the cash market so as to avert a freeze in the credit market, thereby acting as a BOLR.[34] These purchases are financed by issuing *additional* reserves in period 2, over and above the reserves issued in period 1.

Let the subscript b denote the equilibrium under a credible commitment by the central bank to backstop the credit market, and let $\hat{R}_b$ denote the corresponding crisis threshold. The central bank intervenes only in states in which $R < \hat{R}_b$. In those states of nature, the aggregate private supply of loans at rate $r = R$, $L_S^{\text{cred}}(s_b; R)$, exceeds aggregate demand, $L_D^{\text{cred}}(s_b; R)$ —recall Figure 4ii. The central bank can then support the credit market by offering to purchase $S(q; s_b, R)$ business loans in the cash market at price q. The price q is set by the central bank and pins down the equilibrium interbank loan rate, $r = q - 1$ —recall relation (13).

The backstop policy therefore consists of choosing the cash-market price q and the purchase quantity $S(q; s_b, R)$ so as to restore credit-market functioning while maximizing the volume of loans purchased by good banks in that market.

Price. If $q > 1 + R$, all banks will sell their business loans to the central bank only —a highly inefficient outcome— and credit-market functioning will not be restored. Hence, the central bank must set $q \leq 1 + R$. If $q < 1 + R$, some trade will take place in the private credit market. However, the rate of return on interbank loans will not be as high as the rate that good banks can afford to pay: $r < R$. Such a rate will fail to maximize the incentives for bad banks to lend, prompting prospective lenders to reduce borrowers' borrowing limits and demand. Hence, recalling from (19) that $L_D^{\text{cred}}(s_b; r)$ increases with r, the maximum feasible and incentive-compatible demand by good banks is achieved at

$$q = 1 + R.$$

[34]The central bank faces the same asymmetric-information and moral-hazard problems as private lenders: it cannot identify the natural holders of business loans, and the returns on those loans are not pledgeable as collateral. A policy under which the central bank lends beyond the borrowing limit $\bar{\ell}(s; r)$—recall (18)—so that banks can purchase business loans in the cash market would therefore also channel funds to bad banks. Such lending could, in principle, improve the allocation of business loans in a crisis. But it would also expose the central bank to default risk by relaxing the very borrowing constraint that prevents bad banks from mimicking good banks. When defaults are sufficiently costly to lenders, for example because of bankruptcy or debt-collection proceedings, indiscriminate lending is inefficient. For clarity, we abstract from direct lending policies and focus on BOLR interventions in which the central bank purchases business loans itself. For a discussion on the existence of pooling equilibria in a similar environment, where both good and bad banks borrow, see Boissay et al. (2026, online appendix, Section J).

Quantity. At this price, the residual excess supply of loans in the credit market is $L_S^{\text{cred}}(s_b; R) - L_D^{\text{cred}}(s_b; R)$. The central bank can therefore restore credit-market functioning if $S(s_b; R) \geq L_S^{\text{cred}}(s_b; R) - L_D^{\text{cred}}(s_b; R)$. However, purchases beyond the residual excess supply would crowd out private trades and leave an unnecessarily large volume of business loans on the central bank's balance sheet. Therefore, the optimal quantity of loans purchased by the central bank is

$$S(\bar{s}_b; R) = L_S^{\text{cred}}(\bar{s}_b; R) - \underbrace{(1-\mu)\bar{\ell}(\bar{s}_b; R)}_{\substack{\text{maximum} \\ \text{incentive-compatible} \\ \text{demand (from (19))}}}.$$

In effect, the central bank absorbs only the excess supply of business loans needed for the private credit market to clear at $r = R$. Using relations (15) and (19), and denoting average reserve holdings across banks by $\bar{s}_b$, we obtain

$$S(\bar{s}_b; R) = \mu - \bar{s}_b - (1-\mu)(1-\bar{s}_b)R, \quad \forall R < \hat{R}_b. \tag{42}$$

Losses. Because the central bank cannot monitor firms, its purchases of business loans generate losses. To purchase one business loan, the central bank issues $q = 1 + R$ units of reserves, thereby promising to deliver $1 + R$ units of the consumption good in period 3. The loan, however, yields only one unit of the consumption good when held by the central bank. The central bank therefore incurs a loss of R per business loan purchased, reflecting a form of "negative carry". Denoting the central bank's profit by $\Pi^{\text{cb}}(\bar{s}_b; R)$, we have the following result.

Theorem 5 (**Central Bank Losses**). *The backstop policy results in losses for the central bank:*

$$\Pi^{cb}(\bar{s}_b; R) = -RS(\bar{s}_b; R). \tag{43}$$

The central bank's profits, or losses, are assumed to be rebated lump-sum to households.

With the backstop in place, banks are fully insured against crisis states, and their payoffs coincide with those prevailing in normal times, $\Pi^n(s_b; R)$ —recall relation (23). Social welfare under the backstop is therefore given by:[35]

$$\begin{aligned} W_b(s_b; \hat{R}_b) =& (1-s_b)(1+\rho) + s_b - \phi + E(\Pi^{\text{cb}}(\bar{s}_b; R)) \\ =& (1-s_b)(1+\rho-\kappa) + s_b \\ & \underbrace{-(\mu - \bar{s}_b)\Psi(\hat{R}_b) + \overbrace{(1-\mu)(1-\bar{s}_b)\int_0^{\hat{R}_b} R^2 f(R)dR}^{\text{backstop's net benefit} > 0}}_{\text{central bank profit, } E(\Pi^{\text{cb}}(\bar{s}_b;R))<0}. \end{aligned} \tag{44}$$

[35]The expression in (44) comes from the relation $W_b(s_b; \hat{R}_b) = E(\Pi^n(s_b; R) + \Pi^{\text{cb}}(\bar{s}_b; R)) = \int_0^{+\infty} \Pi^n(s_b; R)f(R)dR - \int_0^{\hat{R}_b} RS(\bar{s}_b; R)f(R)dR$, where $S(\bar{s}_b; R)$ is given in (42).

Here $E(\Pi^{\text{cb}}(\overline{s}_b; R))$ denotes the expected lump-sum transfer of central bank profits, or losses, to households. The expression for $W_b(s_b; \hat{R}_b)$ in (44) parallels that for $W(s; \hat{R})$ in (25), except for the last additional term, which captures the net benefit of backstopping the credit market and is strictly positive, all else being equal. Hence, for any given level of reserves $s \in [0, \mu]$,

$$W_b(s, \hat{R}(s)) > W(s, \hat{R}(s)). \tag{45}$$

Optimal Demand for Reserves Under Backstop Policy. Since banks take the lump-sum transfer $E(\Pi^{\text{cb}}(s_b; R))$ as given, their private expected payoff is decreasing in reserve holdings s_b. The following result obtains.

Theorem 6 (**Moral Hazard**). *A credible commitment by the central bank to fully backstop the credit market induces banks to demand no reserves in period 1:*

$$s_b = 0. \tag{46}$$

Theorem 6 is a pure moral-hazard result. Because the backstop insulates banks from the consequences of crisis states, banks have no incentive to hold precautionary reserves. In the absence of such reserves, the crisis threshold is higher and crises occur more frequently. In the aggregate, $\overline{s}_b = s_b = 0$, and relations (21) and (42) become

$$\hat{R}_b = \frac{\mu}{1-\mu}$$

and

$$S(0; R) = \mu - (1-\mu)R.$$

Welfare Outcome Under Backstop Policy. Using (44) and the preceding analysis, welfare under the backstop policy is

$$W_b = 1 + \rho - \kappa - \int_0^{\frac{\mu}{1-\mu}} R(\mu - (1-\mu)R) f(R) dR,$$

which can be rewritten as

$$W_b = \underbrace{1 + (1-\mu)(\rho - \kappa)}_{W^\star} + \chi_b, \tag{47}$$

where

$$\chi_b \equiv (1-\mu) \int_0^{\frac{\mu}{1-\mu}} R^2 f(R) dR - \mu \left[\Psi\left(\frac{\mu}{1-\mu}\right) - \rho + \kappa \right]. \tag{48}$$

The welfare expression in (47) is analogous to those obtained for the laissez-faire economy, (33), and for the economy under optimal regulation, (40); it differs only through the final term. In particular, (33) contains no such term, whereas (40) features χ_r instead.

The main benefit of the backstop policy relative to laissez-faire is that it confines the inefficient holding of business loans in period 2 —in this case by the central bank— to crisis states. The backstop policy is therefore more attractive when the opportunity cost of holding reserves, $\rho-\kappa$, is high. At the same time, because banks hold no reserves in period 1 under the backstop policy, $s_b=0$, the probability of entering a crisis state is higher. The welfare effect relative to laissez-faire is therefore ambiguous. Relations (33) and (47) imply that the backstop policy is welfare-improving, that is, $W_b > W^\star$, if and only if

$$\chi_b > 0$$

or, equivalently,

$$\rho-\kappa > \Psi\left(\frac{\mu}{1-\mu}\right) - \frac{1-\mu}{\mu}\int_0^{\frac{\mu}{1-\mu}} R^2 f(R)dR, \tag{49}$$

where both sides are positive. The backstop policy is desirable only when the period-1 opportunity cost of holding reserves, captured by the left-hand side, is sufficiently high. Equivalently, this opportunity cost must exceed the return-at-risk, $\Psi(\mu/(1-\mu))$, net of the expected benefit from reallocating business loans in crisis states, captured by the second term on the right-hand side. In particular, when the return-at-risk, $\Psi(\mu/(1-\mu))$, is close to the expected net return on loans, $\rho-\kappa$, backstopping the credit market improves welfare relative to laissez-faire. When condition (49) is not satisfied, however, the backstop may *reduce* welfare compared to laissez-faire. This outcome can be interpreted as a liquidity paradox: by reducing private liquidity holdings *ex ante*, the prospect of public liquidity provision in bad states may reduce aggregate liquidity overall and, ultimately, welfare. A similar —but distinct— paradox is described in, *e.g.*, Choi et al. (2016) and Acharya and Rajan (2024).

We next compare the backstop policy with optimal liquidity regulation. Using $\overline{s}_b = s_b$ and substituting into (44) the expression for s_b in terms of $\hat{R}_b$ from (22), we obtain

$$W_b(s(\hat{R}_b),\hat{R}_b) = 1+(1-\mu)(\rho-\kappa)+\frac{(1-\mu)^2\hat{R}_b}{1-(1-\mu)\hat{R}_b}\left[\rho-\kappa-\Psi(\hat{R}_b)+\frac{1}{\hat{R}_b}\int_0^{\hat{R}_b} R^2 f(R)dR\right].$$

This expression is increasing in $\hat{R}_b$ over the feasible range $\hat{R}_b \le \frac{\mu}{1-\mu}$, and therefore decreasing in s_b for $s_b \ge 0$, if

$$\rho-\kappa \ge \Psi\left(\frac{\mu}{1-\mu}\right) - (1-\mu)\int_0^{\frac{\mu}{1-\mu}} R^2 f(R)dR. \tag{50}$$

When inequality (50) holds, $W_b(0,\mu/(1-\mu)) > W_b(s_r,\hat{R}(s_r))$. Moreover, since $W_b(s_r,\hat{R}(s_r)) > W(s_r,\hat{R}(s_r))$ per (45), condition (50) guarantees that $W_b(0,\mu/(1-\mu)) > W(s_r,\hat{R}(s_r))$, which means that the backstop policy, with $s_b=0$, delivers higher welfare than optimal liquidity regulation, with $s_r>0$. Condition (50) is stronger than condition (49). The following theorem summarizes the welfare comparison.

Theorem 7 (Welfare Comparison). *Under the backstop policy, welfare is higher than under laissez-faire, that is, $W_b > W^\star$, if and only if*

$$\rho - \kappa > \Psi\left(\frac{\mu}{1-\mu}\right) - \frac{1-\mu}{\mu}\int_0^{\frac{\mu}{1-\mu}} R^2 f(R)dR.$$

Moreover, welfare under the backstop policy is higher than under optimal liquidity regulation, that is, $W_b > W_r$, if

$$\rho - \kappa > \Psi\left(\frac{\mu}{1-\mu}\right) - (1-\mu)\int_0^{\frac{\mu}{1-\mu}} R^2 f(R)dR.$$

4.3 Policy Mix

The policy mix consists of a minimum liquidity requirement, set by the regulator at s_m, and a state-contingent central bank backstop of the credit market. To nest the benchmark regimes in a single formulation, let $\lambda_m \in [0,1]$ denote the probability with which the central bank implements the backstop in crisis states. The subscript m denotes variables evaluated under the policy mix. This formulation nests the relevant benchmark regimes: the case $\lambda_m = 0$ and $s_m \leq s^\star$ corresponds to laissez-faire; the case $\lambda_m = 0$ and $s_m = s_r$ corresponds to liquidity regulation; and the case $\lambda_m = 1$ and $s_m = 0$ corresponds to the plain-vanilla backstop policy.[36]

The regulator then chooses (s_m, λ_m) to maximize welfare, $W_m(s_m, \lambda_m, \hat{R}_m)$:[37]

$$\begin{aligned} W_m(s_m, \lambda_m, \hat{R}_m) =& (1-s_m)(1+\rho-\kappa) + s_m - (\mu - s_m)\Psi(\hat{R}_m) \\ &+ \lambda_m(1-\mu)(1-s_m)\int_0^{\hat{R}_m} R^2 f(R)dR. \end{aligned} \tag{51}$$

The maximization is over s_m and λ_m, subject to the relationship between the crisis threshold $\hat{R}_m$ and the minimum liquidity requirement s_m in (21), assuming that the requirement binds. Under a minimum liquidity requirement, banks are constrained in their reserve holdings: the regulator directly sets s_m, which pins down $\hat{R}_m$ independently of λ_m. It follows that, for any given $s_m < 1$, welfare $W_m(s_m, \lambda_m)$ is strictly increasing in λ_m. Hence, it is optimal to commit to backstopping the credit market:

$$\lambda_m = 1.$$

When $\lambda_m = 1$, banks privately choose not to hold reserves, so any strictly positive minimum liquidity requirement binds; recall Theorem 6. Substituting for s_m using its

[36] Although the optimum will set $\lambda_m = 1$ once s_m is chosen, introducing λ_m is useful because it makes clear that the policy mix nests all these special cases.

[37] In (51), the expression for $W_m(s_m, \lambda_m, \hat{R}_m)$ follows from $W_m(s_m, \lambda_m, \hat{R}_m) = \overline{\Pi}(s_m) + E(\Pi^{cb}(s_m; R)) = \int_0^{\hat{R}_m} [(1-\lambda_m)\Pi^c(s_m; R) + \lambda_m \Pi^n(s_m; R)] f(R)dR + \int_{\hat{R}_m}^{+\infty} \Pi^n(s_m; R) f(R)dR + E(\Pi^{cb}(s_m; R))$, where $\Pi^{cb}(s_m; R) = -R(\mu - s_m - (1-\mu)(1-s_m)R)$ and $\Pi^n(s_m; R)$ and $\Pi^c(s_m; R)$ are given in (23) and (24), respectively.

expression in terms of $\hat{R}_m$ from (22), we can rewrite the regulator's problem as choosing $\hat{R}_m$ to maximize

$$W_m(\hat{R}_m) = 1 + (1-\mu)(\rho - \kappa) \\ + \frac{(1-\mu)^2 \hat{R}_m}{1-(1-\mu)\hat{R}_m} \left[\rho - \kappa - \Psi(\hat{R}_m) + \frac{1}{\hat{R}_m} \int_0^{\hat{R}_m} R^2 f(R) dR \right]. \tag{52}$$

The derivative with respect to $\hat{R}_m$ has the sign

$$W_m'(\hat{R}_m) \geq 0 \quad \Leftrightarrow \quad \Psi_m(\hat{R}_m) \leq \rho - \kappa, \tag{53}$$

where

$$\Psi_m(\hat{R}) \equiv \Psi(\hat{R}) - (1-\mu) \int_0^{\hat{R}} R^2 f(R) dR. \tag{54}$$

Since $\Psi_m'(\hat{R}) > 0$ over the feasible range of crisis thresholds $\hat{R} < \mu/(1-\mu)$, condition (53) characterizes an interior maximum when it holds with equality. In that case, the optimality condition parallels the private first-order condition in (30) and the social first-order condition in (37), except that it involves $\Psi_m(\hat{R}_m)$ rather than $\Psi(\hat{R})$ or $\Psi_r(\hat{R}_r)$. Moreover,

$$\Psi_m(\hat{R}) < \Psi(\hat{R}). \tag{55}$$

It follows that, when the optimum is interior, the optimal policy mix entails a higher crisis threshold than under laissez-faire. Equivalently, it requires a smaller liquidity buffer than the one chosen under laissez-faire:

$$s_m < s^\star.$$

We can now state our central result, the *market backstop principle.*[38]

Theorem 8 (Market Backstop Principle). *The socially optimal policy mix has four features:*

1. *The central bank intervenes only in crisis states in which purchases of illiquid assets —business loans in the model— restore market functioning; that is, only when $R \in (0, \hat{R}_m)$.*

2. *In those states, the central bank purchases exactly the quantity of assets needed to restore trade, $S(s_m; R)$, and no more. Purchases take place at the restored-market price, $q = 1 + R$.*

[38]The market backstop principle remains effective even when the policy framework is expanded to include the central bank paying interest on reserves; see Section H of the online appendix.

3. *Asset purchases are complemented by a minimum liquidity requirement that limits the moral hazard created by expected ex post support. The optimal liquidity requirement is either zero or strictly positive but modest, in the sense that it remains below banks' laissez-faire liquidity holdings:* $s_m < s^\star$*; the optimal mix's requirement is*

$$s_m = \max\left\{0 \;\; ; \;\; \frac{\mu - (1-\mu)\Psi_m^{-1}\left(\rho - \kappa\right)}{1-(1-\mu)\Psi_m^{-1}\left(\rho - \kappa\right)}\right\}. \tag{56}$$

4. *The market backstop entails central-bank losses in crisis states, but these losses are outweighed by the welfare gains from restoring credit-market functioning.*

The backstop principle outlined in Theorem 8 can be viewed as the BOLR analogue of the classical Bagehot principle for LOLR interventions. Both guide central bank action in crises, but they apply to different objects and lead to different prescriptions. Bagehot concerns individual financial institutions: the central bank lends freely to solvent institutions, against sound collateral, and at a penalty rate. The market backstop concerns financial markets: the central bank intervenes parsimoniously —only when trade breaks down, by purchasing only the distressed assets and only in the quantities needed to restore trade— and at the fair price that prevails once the market is functioning again. Such purchases may generate central bank losses, but they are welfare-improving because they restore trade. Moral hazard is addressed not through a penalty rate —or, equivalently, a lower purchase price— but through a minimum liquidity requirement. This requirement strengthens the system-wide demand for central bank reserves in normal times, in anticipation of possible stress, and is set at the level needed to contain the moral hazard created by the prospect of intervention —no more and no less.

By reducing the cost of credit-market freezes, the backstop policy weakens the case for precautionary reserves and leads the regulator to lower the minimum liquidity requirement. With a backstop, banks need not self-insure as much *ex ante*, because the central bank can restore market functioning *ex post*. The reserve requirement is therefore used primarily to discipline banks and contain moral hazard, rather than to substitute for missing market liquidity. Consequently, the optimal requirement is lower than both the requirement that would prevail in the absence of a backstop, $s_m < s_r$, and banks' laissez-faire reserve holdings, $s_m < s^\star$, and may even be zero.

The optimal policy mix entails a corner solution with no minimum liquidity requirement, $s_m = 0$, when the opportunity cost of reserves is sufficiently high relative to the adjusted return-at-risk, that is, when

$$\Psi_m\left(\frac{\mu}{1-\mu}\right) < \rho - \kappa. \tag{57}$$

In this case, welfare is increasing in $\hat{R}_m$ —recall (53)— and, equivalently, decreasing in s_m for all $s_m \geq 0$. When inequality (57) holds, the period-1 opportunity cost of requiring banks to hold reserves, $\rho - \kappa$, exceeds the welfare gain from inducing additional loan reallocation, as summarized by $\Psi_m\left(\frac{\mu}{1-\mu}\right)$. The optimal policy mix therefore sets $s_m = 0$. In this case, the regulatory requirement coincides with banks' private demand for reserves under the backstop —recall that $s_b = 0$— and the optimal policy mix collapses to the plain-vanilla backstop policy studied in Section 4.2.

Welfare Outcome Under Policy Mix. Using (52), (53), and (54), welfare under the optimal policy mix can be written as

$$W_m = \underbrace{1 + (1-\mu)(\rho - \kappa)}_{W^{\star}} + \chi_m, \tag{58}$$

where

$$\chi_m \equiv \max\left\{(1-\mu)^2 \int_0^{\hat{R}_m} R^2 f(R) dR \; ; \; \chi_b\right\}. \tag{59}$$

Expression (58) parallels the welfare expressions obtained in the absence of policy, (33), under socially optimal liquidity regulation, (40), and under the backstop policy, (47); it differs only through the policy-specific term χ_m. The definition of χ_m in (59) reflects the possibility that the optimal policy mix may dispense with liquidity requirements altogether when the period-1 opportunity cost of holding reserves is sufficiently high. In that case, the optimal policy mix coincides with the plain-vanilla backstop policy. Theorem 9 follows because the policy mix nests both benchmark interventions: liquidity regulation and the backstop policy.

Theorem 9 (**Welfare Comparison**)**.** *Welfare is weakly higher under the optimal policy mix than under any benchmark regime:*

$$W_m \geq \max\left\{W_r \; ; \; W_b \; ; \; W^{\star}\right\}.$$

5 Numerical Illustration

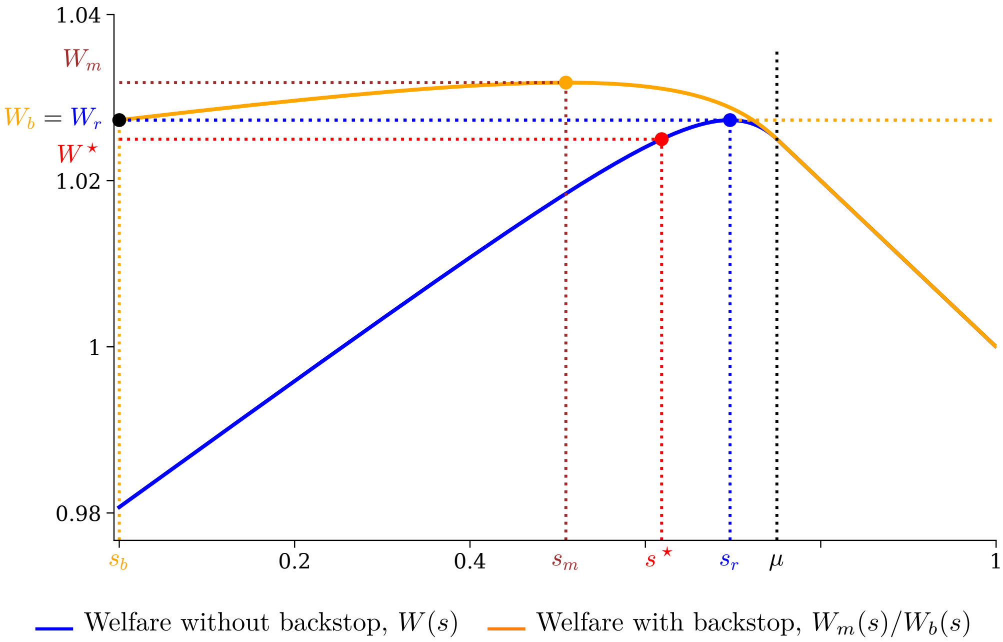
This section is not intended to provide a quantitative evaluation of policy interventions. Its purpose is instead illustrative: to clarify the central policy trade-off in the model. Requiring banks to hold central bank reserves *ex ante*, in period 1, helps reduce the likelihood of a credit-market freeze in period 2 and the associated expected crisis losses. Reserves, however, are costly: they absorb balance-sheet capacity that could otherwise be used to finance productive business loans. The analysis below shows how this trade-off shapes the desirability of liquidity regulation, central-bank backstops, and their optimal combination.

For the numerical illustration, we arbitrarily set the fraction of bad banks to $\mu = 0.75$ and the loan-origination cost to $\kappa = 0.1$. For the density $f(R)$, we use a Gamma distribution, which is sufficiently flexible to generate variation in both the expected return, ρ, and the return-at-risk, RaR:[39]

$$f(R;k,\theta)=\frac{R^{k-1}e^{-\frac{R}{\theta}}}{\theta^k\Gamma(k)}. \tag{60}$$

In the baseline parametrization, we choose the distribution parameters k and θ so that $\rho=0.2$ and $W_r=W_b$. The baseline therefore describes an economy in which the optimal minimum liquidity requirement delivers the same welfare as the plain-vanilla backstop policy. In this parametrization, return-at-risk is RaR ≈ 0.16.[40]

Figure 5: Policies and Welfare Outcomes



Notes: Plot of the welfare functions $W(s)$ and $W_m(s)/W_b(s)$ for $\kappa=0.1$ and $\mu=0.75$. The parameters of the Gamma distribution, k and θ, are set so that $\rho=0.2$ and $W_r=W_b$, implying RaR ≈ 0.16.

Figure 5 plots the welfare schedule without a backstop, $W(s)$, and the welfare schedule with a backstop, $W_m(s)$ —or $W_b(s)$, as functions of banks' reserve holdings in period 1. The figure illustrates the central trade-off between liquidity insurance and the crowding out of productive lending. Both welfare schedules are hump-shaped. At low levels of reserves, increasing s improves welfare because it lowers the crisis threshold, reduces the probability of a credit-market freeze in period 2, and decreases the associated expected crisis loss.

[39] For a graphical illustration of the Gamma density under the chosen parameters, see Section E of the online appendix.

[40] Note that RaR $\equiv \Psi\left(\frac{\mu}{1-\mu}\right)=\rho\int_0^{\frac{\mu}{1-\mu}}\frac{R^k e^{-R/\theta}}{\theta^{k+1}\Gamma(k+1)}dR=\rho\cdot F\left(\frac{\mu}{1-\mu};k+1,\theta\right)$, where $F(\cdot,k+1,\theta)$ is the cumulative distribution function of a Gamma distribution with shape parameter $k+1$ and scale parameter θ. Given $\rho=k\theta$ and μ, one needs to solve numerically for k and θ so that $\rho=0.2$ and $W_b=W_r$, and then recover the corresponding value of RaR.

Beyond some point, however, additional reserves become socially costly: they absorb balance-sheet capacity that could otherwise be used to originate productive business loans in period 1. The declining portion of each welfare schedule reflects this opportunity cost.

The comparison between the two curves highlights the role of the backstop. For any given reserve buffer, welfare is higher with the backstop than without it, because central-bank purchases mitigate the cost of crisis states by restoring credit-market functioning —recall relation (45). Moreover, the welfare curve is flatter under the backstop. This reflects the fact that, once the central bank can intervene in crisis states, welfare becomes less sensitive to banks' *ex ante* reserve holdings. A smaller reserve buffer still raises the likelihood of a crisis, but the social cost of such a crisis is lower because the backstop limits the inefficient disruption of credit-market trade.

The figure also clarifies why liquidity regulation and backstops can be substitutes. In the absence of a backstop, welfare maximization calls for a relatively large reserve buffer, since reserves are the only instrument available to reduce expected crisis losses. With a backstop, by contrast, part of the insurance otherwise provided by reserves is supplied *ex post* by the central bank. The optimal reserve requirement is therefore lower —recall Theorem 8. In the baseline parametrization, the parameters are chosen so that the welfare gain from optimal liquidity regulation, evaluated at $s = s_r$, exactly equals the welfare gain from the plain-vanilla backstop, evaluated at $s = s_b = 0$. This benchmark is useful: it describes an economy in which *ex ante* liquidity insurance and *ex post* market support are equally desirable, while also illustrating why their optimal combination may dominate either policy in isolation.

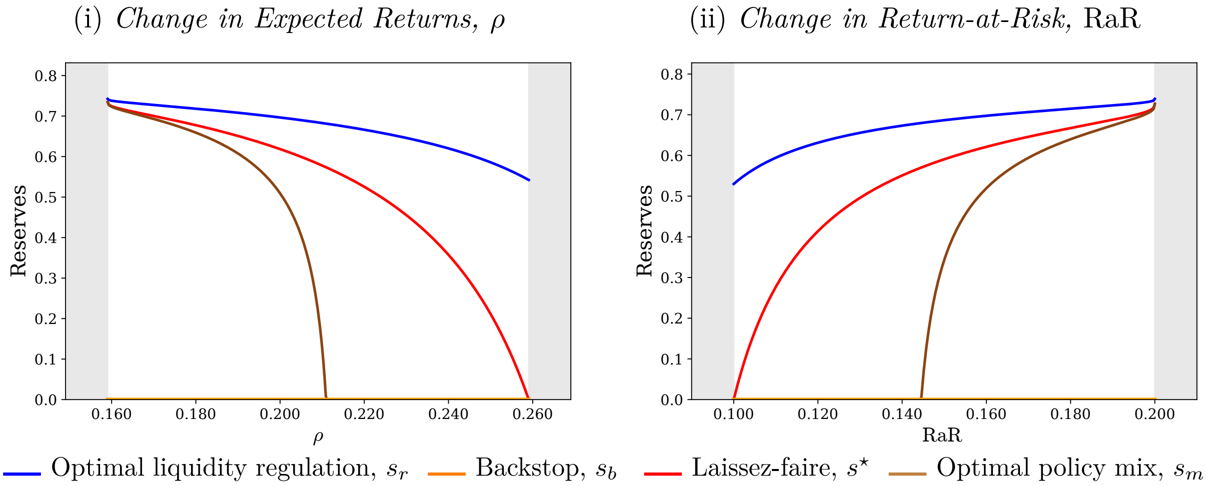
6 Discussion

6.1 Economic Fundamentals and Optimal Policy

We now examine how the optimal policy mix varies with the underlying economic fundamentals. In the model, these fundamentals are summarized by two statistics: the expected return on business loans, ρ, and return-at-risk, RaR. The first governs the opportunity cost of reserves; the second summarizes the expected cost of credit-market freezes. Figure 6 shows how reserve holdings under the different policy regimes respond to changes in these fundamentals, while Figure 7 summarizes the resulting policy regions in the (ρ, RaR) space.

Figure 6 reports the comparative statics of reserve holdings. Panel 6i varies the expected excess return ρ while holding return-at-risk fixed. As ρ rises, all policies that require *ex ante* liquidity become less attractive, and the optimal reserve buffer declines.

Figure 6: Reserve Holdings and Economic Fundamentals under Varied Policies



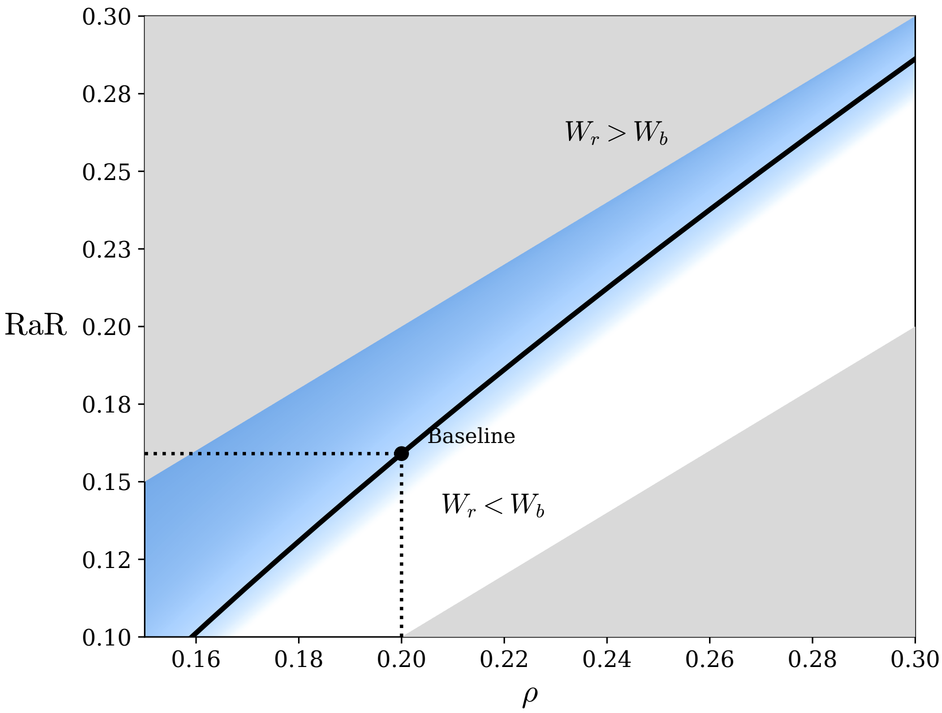
Notes: Banks' reserves holding computed for $\kappa = 0.1$ and $\mu = 0.75$. The parameters of the Gamma distribution, k and θ, are set so that RaR ≈ 0.16 and $\rho \in (\text{RaR}, \text{RaR} + \kappa)$ in panel 6i, and so that $\rho = 0.2$ and RaR $\in (\rho - \kappa, \rho)$ in panel 6ii. The gray area is the set of parameter values that are either infeasible (RaR $\geq \rho$) or that violate Assumption 1 (RaR $\leq \rho - \kappa$). For a similar analysis of welfare, refer to Section F in the online appendix.

Panel 6ii varies return-at-risk while holding ρ fixed. As crisis risk rises, precautionary liquidity becomes more valuable, and the optimal reserve buffer increases whenever the liquidity requirement is positive. The plain-vanilla backstop remains associated with zero private reserve holdings, $s_b = 0$, reflecting the moral-hazard effect identified in Theorem 6. The policy mix, by contrast, combines the backstop with a positive liquidity requirement once return-at-risk is sufficiently high.

The same trade-off is summarized in Figure 7. The black line is the policy frontier: it gives the combinations of ρ and RaR for which optimal liquidity regulation and the plain-vanilla backstop deliver the same welfare, $W_r = W_b$. Below the frontier, the backstop dominates liquidity regulation. Above the frontier, liquidity regulation becomes relatively more valuable. In the white region, the optimal liquidity requirement is zero, $s_m = 0$, so the policy mix coincides with the plain-vanilla backstop. In the blue region, the optimal policy mix combines a backstop with a positive minimum liquidity requirement. The shading describes the optimal policy mix: darker shades correspond to higher values of s_m.

The figure highlights that the optimal policy is state-dependent. When investment opportunities are strong and crisis risk is moderate, it is efficient to rely primarily on *ex post* market support. When crisis risk is sufficiently high, a backstop alone generates excessive reliance on public support, and a positive liquidity requirement becomes optimal. The role of liquidity regulation in the policy mix is therefore not to replace the backstop, but to discipline banks' reserve choices and reduce the cost of *ex post* interventions.

Figure 7: Policy Frontier and Optimal Policy Mix



Notes: The optimal-policy frontier, shown by the black line, corresponds to the combinations of ρ and RaR for which welfare under optimal liquidity regulation (W_r) is equal to welfare under plain-vanilla backstop (W_b). Below the frontier, $W_r < W_b$. The shade of blue indicates the level of the minimum liquidity requirement in the optimal policy mix, s_m: darker shades correspond to higher values of s_m. In the white region, $s_m = 0$, and the policy mix coincides with the backstop policy, so that $W_m = W_b$. The upper gray region contains infeasible parameters (RaR $\geq \rho$). The lower gray region contains parameters that violate Assumption 1 (RaR $\leq \rho - \kappa$). The black dot corresponds to the baseline parametrization used in Figure 5.

This state dependence should not be interpreted as a prescription for either pro-cyclical or counter-cyclical liquidity regulation. All else equal, the analysis suggests that liquidity requirements are more costly during economic booms, when investment opportunities are strong and the expected return ρ is high. But the optimal requirement also depends on the tail risk associated with the boom. If strong investment opportunities coincide with only moderate crisis risk, the opportunity cost of liquidity buffers outweighs their benefits, and the optimal policy relies mainly on the *ex post* backstop. By contrast, if the boom is accompanied by sufficiently large tail risk —as captured by a high RaR— a higher liquidity requirement may be warranted despite its opportunity cost. Thus, the policy prescription is neither simply pro-cyclical nor counter-cyclical. Rather, it depends on the interplay between the strength of investment opportunities and the tail risk building up in the background.

6.2 Central Bank Balance Sheet: Size and Composition

We next examine how liquidity regulation affects the expected size, composition, and losses of the central bank's balance sheet. In this subsection, let s_s denote a generic minimum liquidity requirement. Given s_s, the central bank purchases $S(s_s; R)$ loans at price $q = 1 + R$ in states of nature $R < \hat{R}(s_s)$. Let $E(B^{\text{cb}})$ denote the expected size of the central bank's balance sheet, and let $E(\Pi^{\text{cb}})$ denote expected central bank profits, across all states of nature. Formally,

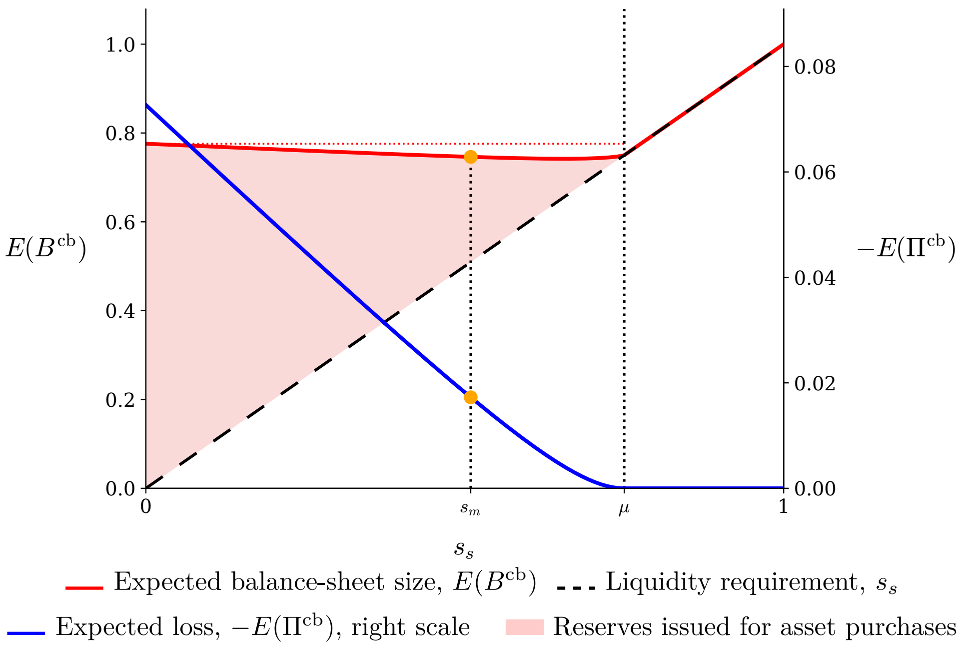
$$E(B^{\text{cb}}) \equiv s_s + \int_0^{\hat{R}(s_s)} (1+R) \underbrace{[\mu - s_s - (1-\mu)R(1-s_s)]}_{\text{purchases of business loans } S(s_s;R)} f(R)dR,$$

where the first term on the right-hand side, s_s, corresponds to the reserves issued in period 1 against stored goods, and the second term corresponds to the reserves issued in period 2 against business loans. Expected central bank profits are

$$E(\Pi^{\text{cb}}) \equiv -\int_0^{\hat{R}(s_s)} R\,[\mu - s_s - (1-\mu)R(1-s_s)]\, f(R)dR.$$

Expected central bank losses are therefore $-E(\Pi^{\text{cb}})$.

Figure 8: Central Bank's Expected Size $E(B^{\text{cb}})$ and Losses $-E(\Pi^{\text{cb}})$



Notes: Plot of $E(B^{\text{cb}})$ and $-E(\Pi^{\text{cb}})$ at the policy mix, for $s_s \in [0, 1]$. The orange dot corresponds to the optimal policy mix, as in Figure 5. For values of $s_s \geq \mu$, the credit market is not needed and the backstop policy is not relevant; in that case, $B^{\text{cb}}(s_s) = s_s$ and $E(\Pi^{\text{cb}}) = 0$.

The relationship between liquidity requirements and the expected size of the central bank balance sheet is ambiguous because a minimum liquidity requirement has two opposing effects. On the one hand, a higher reserve buffer mechanically increases the central bank balance sheet *ex ante*, in period 1. On the other hand, it reduces both the frequency and the scale of credit-market backstops in period 2. Since these backstops are funded by issuing additional reserves, a stricter liquidity requirement tends to reduce the central bank balance sheet across crisis states.[41]

Figure 8 illustrates these two forces for the baseline parametrization of the model. For low values of s_s, increasing the liquidity requirement can reduce the expected size of the central bank balance sheet, shown by the red line. The reason is that the indirect effect dominates: a larger reserve buffer *ex ante* makes crises less frequent and reduces the volume of business loans that the central bank will have to purchase *ex post*, when crises occur (shaded area). It also increases welfare, as shown in Figure 5 by the orange line.

The figure also shows that expected losses decline with the liquidity requirement (blue line). Period-1 reserve issuance mechanically increases the size of the central bank balance sheet, as shown by the dashed line, but it does not generate losses for the central bank. Losses arise instead from purchases of business loans during crises, represented by the shaded area. Thus, a larger period-1 balance sheet may entail lower losses than a smaller balance sheet that is repeatedly expanded through backstop interventions in period 2.[42]

7 Conclusion

Market freezes have become a prominent form of financial instability in modern financial systems. This paper has therefore focused on BOLR interventions. This focus matters because market freezes call for a different policy response from traditional bank runs. In a bank run, the central question is how the central bank should support a solvent institution facing a funding shock. In a market freeze, the question is how public liquidity should restore trade when assets fail to reach their natural holders. The market backstop principle reflects this distinction. It calls for the central bank to purchase the distressed assets rather than lend against collateral, to pay the restored-market price rather than charge a penalty rate, and to contain moral hazard through liquidity regulation rather than through the terms of the intervention itself.

Our analysis characterizes this principle in a model of secondary securities markets prone to freezes. The model deliberately abstracts from the specificities and details of historical episodes to focus on the core mechanisms, frictions, and policy trade-offs. Trade

[41] In a static model like ours, the states of nature can be interpreted as representing different points in time. Repeated backstop interventions across these states of nature (or over time) often lead to a ratchet effect, gradually increasing the size of the central bank, as illustrated in Figure G.1.

[42] Section G in the online appendix illustrates that each major episode of market stress in the U.S. was followed by a sharp upward shift in reserves, while subsequent normalization remained only partial.

can be disrupted by private information and moral hazard, preventing assets from reaching their natural holders. Central bank reserves play a crucial role in mitigating this failure by supporting trade when private credit markets freeze.

The market backstop principle follows from the optimal policy mix. The central bank intervenes only in crisis states, purchases only what is needed to restore market functioning, and pays the price that prevails once trade is restored. Because such interventions create moral hazard, they should be complemented by a modest minimum liquidity requirement.

The same trade-off also governs the central bank balance sheet. Higher reserve requirements enlarge the balance sheet *ex ante*, but can reduce the frequency and scale of crisis interventions. Leaner balance sheets, by contrast, may ultimately become larger if recurrent market freezes force repeated emergency asset purchases. Reserves held before crises and reserves issued during crises are therefore two sides of the same policy problem: how public liquidity should be supplied in a financial system exposed to market freezes.

Several questions remain open. Under the backstop principle, the central bank purchases assets from institutions that have access to its deposit facility and comply with liquidity regulation. In practice, however, market freezes may involve institutions that lack access to central bank reserves, operate outside the regulatory perimeter, or both, as illustrated in Table A. Our analysis therefore points to the possibility that BOLR interventions could be more effective if relevant core non-bank financial institutions were granted access to central bank deposit facilities, together with appropriate liquidity requirements.[43] We have also abstracted from central bank lending as a tool for market support. Recent crises suggest that such lending can also play an important role in restoring market functioning. Extending the analysis to incorporate central bank lending would help clarify how LOLR facilities, BOLR interventions, and liquidity regulation should be combined in a broader financial system.

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[43] An example of such a facility is the Federal Reserve's Overnight Reverse Repo Facility, which is available to MMFs. In parallel, international standard-setting bodies and certain national regulatory authorities have recently proposed minimum liquid asset requirements for MMFs (FSB, 2021; FCA, 2026).

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Appendix

Table A: Secondary-Market Freezes: A Three-Episode Comparison

Constrained-absorption pattern	Global Financial Crisis (2007–09)	Global dash for cash (March 2020)	UK gilt/LDI crisis (Sept.–Oct. 2022)	Model Mapping
1. Distressed financial assets	Commercial paper, non-agency RMBS/structured ABS, corporate bonds.	Long-dated U.S. Treasuries, agency MBS, investment-grade/high-yield corporate bonds.	Long-maturity gilts.	Business loans come under stressed when their excess return R falls below the crisis threshold $\hat{R}$
Triggers	Rising subprime delinquencies, house-price declines, and mortgage losses first distressed structured credit; Lehman's failure and the Reserve Primary Fund's breaking of the buck then triggered a run on MMFs and a generalized withdrawal of wholesale funding.	The abrupt pandemic/lockdown shock produced extreme uncertainty and a global dash for dollars. Investors tried to monetize normally liquid securities simultaneously. For many safe assets, the impairment reflected market liquidity rather than fundamentals.	The announcement of the U.K. "mini-budget" — large unfunded tax cuts and prospective extra gilt issuance — caused a sharp reassessment of fiscal/inflation risk and an exceptional rise in long gilt yields.	An adverse aggregate shock drives R below $\hat{R}$. The causes are not modeled explicitly.
2. Forced/natural sellers	MMFs, SIVs, and ABCP conduits.	Prime MMFs, leveraged hedge funds, and non-U.S. foreign investors.	Leveraged LDI funds acting for defined-benefit pension schemes.	The mass μ of bad banks, whose excess return on business loans is lower than that of good banks, are the natural sellers.
Reasons for forced sales	Redemptions, precautionary cash demand, funding withdrawals, and losses.	Redemptions, precautionary cash demand, FX-reserve needs, margin calls, and VaR/risk-limit reductions.	Rising risk premia on long-term gilts and lower gilt prices.	Bad banks' zero excess return is a reduced-form shortcut for the forces behind forced sales — *e.g.*, deficient monitoring, shifts in clientele's preferred habitat, redemptions — which are not modeled explicitly.
3. Constrained natural buyers	Commercial banks were the principal private net buyers of securitized assets.	At peak stress, primary dealers were the principal immediate private buyers, taking customer sales of Treasuries, agency MBS, and corporate bonds onto inventory.	Hedge funds and gilt-edged market makers were the effective private buyers.	The mass $1-\mu$ of good banks, whose excess return on business loans is higher than that of bad banks, are the natural buyers.
Reasons for limited absorption capacity	The correlated mortgage and funding shock made the usual liquidity providers (*e.g.* hedge funds, broker-dealers) net sellers, leaving commercial banks as the main "residual" private marginal buyers. Banks increased holdings substantially, but their capacity was limited by equity risk-capital. The effective private buyer base was therefore narrow relative to the assets for sale.	Customer selling accumulated on dealer inventories because many usual end-buyers were themselves raising cash or unwinding leverage. Dealer balance sheets were finite, while volatility raised VaR, hedging costs, margins, and balance-sheet charges.	Private absorption was concentrated in dealer inventories and a handful of hedge funds. Dealer inventory and risk limits bound; hedge funds had finite leverage and risk capital. Insurers and non-LDI asset managers had similar long-duration exposure, further narrowing the immediately available buyer base.	Bad banks' zero excess return encourages them to mimic good banks by borrowing and then absconding in search for yield. When $R < \hat{R}$, the moral-hazard friction becomes material: good banks' borrowing constraint binds, and they cannot absorb bad banks' entire business-loan portfolio. The higher is μ, the thinner is the natural-buyer base.

Notes: This table reviews historical secondary-market freeze episodes along three dimensions: (1) the type of distressed financial assets, (2) the type of forced or natural sellers, and (3) the type of natural buyers and the balance-sheet constraints that prevented them from absorbing the full volume of forced sales. The table is based on Duygan-Bump et al. (2013); He et al. (2010); Merrill et al. (2021); Dick-Nielsen et al. (2012) for 2007–09; Fleming et al. (2021); Duffie (2020); Duffie et al. (2023); Kargar et al. (2021); O'Hara and Zhou (2021) for March 2020; and Alexander et al. (2023); Pinter (2023); Pinter et al. (2024) for the gilt episode. The comparison shows that, although the distressed assets and relevant institutions differed across episodes, a similar constrained-absorption structure was present in each case. The last column maps the components of our model into this constrained-absorption pattern, motivating a broad interpretation of the terms "banks" and "business loans".

Supplemental Material
for Online Appendix

A A Preferred-Habitat Interpretation

The model can also be interpreted as a reduced-form model of preferred-habitat trading, following the classic preferred-habitat view of Modigliani and Sutch (1966) and its modern formalization by Vayanos and Vila (2021).

Under this interpretation, the relevant agents are not banks holding business loans, but institutional asset managers —henceforth "funds"— holding financial securities on behalf of their clients and shareholders. Clients and shareholders may value assets not only for their expected pecuniary returns, but also for how well they fit their liabilities, mandates, liquidity needs, or clienteles at a given point in time. Pension funds may have a natural demand for long-term bonds, money-market funds for short-term safe instruments, insurers for assets with maturities matching expected claims, and banks for liquid short-term claims. Such preferences create natural buyers and sellers for particular asset classes or maturity segments.

To map this interpretation into the model, relabel banks as mutual funds and bank depositors as mutual-fund shareholders. In period 1, funds invest in two assets: financial securities and central bank reserves. Both securities and reserves pay one unit of the consumption good in period 3. However, unlike reserves, securities may also provide additional non-pecuniary clientele benefits.

In period 2, mutual-fund shareholders receive idiosyncratic preference shocks. The shareholders of a fraction $1 - \mu$ of funds come to prefer securities to reserves: for them, each unit of securities yields an additional utility benefit equivalent to R consumption goods in period 3. We call these funds *specialists*, since their shareholders value the specific exposure provided by the securities. The shareholders of the remaining fraction μ are indifferent between securities and reserves, as the two assets have the same pecuniary payoff. We call these funds *generalists*. Under this interpretation, welfare is measured by the expected utility delivered by funds' asset portfolios, rather than by expected pecuniary payoffs alone. The preferences of a fund's shareholders are assumed to be private information known exclusively to that fund. Consequently, it is not possible to differentiate specialist funds from generalist funds.[1]

Formally, the model and analysis are unchanged. The period-2 idiosyncratic shocks create gains from reallocation. Before preferences are realized, securities are held equally by specialists and generalists. Once preferences are realized, however, securities create

[1]We are considering a scenario where asset managers cater to specific types of investors and may deduce their preferences based on characteristics such as whether they are retail or institutional investors, their size (large or small), and the categories of funds they choose, such as government, prime, or tax-exempt funds (see Johnson (2004)).

greater value in the hands of specialists. Specialists are therefore the natural holders of the securities, while generalists are the natural sellers. Efficient trade requires securities to be reallocated from generalist funds to specialist funds.

This interpretation gives the excess return R a broader meaning. It need not represent a technological return from monitoring business loans. Instead, it captures the surplus created when financial assets are held by their natural holders. This surplus may arise because specialists manage the assets more effectively, because their shareholders derive greater utility from the assets' payoff structure, or because the assets better match their institutional constraints —in line with notion of "preferred-habitat". In this sense, "business loans" in the baseline model stand more generally for financial claims whose secondary-market liquidity —and hence reallocation— may become impaired during crises.

The role of reserves is unchanged. Reserves allow specialist funds to purchase securities from generalist funds in the cash market. When specialists hold enough reserves, securities can be reallocated even if private credit is impaired. When reserves are scarce, specialists must rely on credit to finance their purchases. But if lenders cannot distinguish specialists from generalists, and if generalists can mimic specialists in order to borrow and default, credit may be rationed. The secondary securities market may then freeze, leaving securities with generalists even though specialists value them more.

B Safe Assets *versus* Reserves, and the Shadow Value of Reserves

This section shows that, in the context of the model, banks strictly prefer central bank reserves to safe assets. In the baseline model, banks invest $1 - s$ in business loans and s in storage, which we refer to as the "safe asset". Suppose now that, in period 1, banks may sell a fraction ν of their safe assets to the central bank at price p, expressed in units of reserves. Banks therefore exchange νs units of safe assets for $p\nu s$ units of central bank reserves, while retaining $(1 - \nu)s$ units of safe assets on their balance sheets. Both safe assets and reserves pay one unit of the consumption good in period 3. Their only difference is that reserves can be used as a means of settlement in the period-2 cash market, whereas safe assets cannot.[2] Thus, when banks enter period 2, their reserve holdings $p\nu s$ provide the liquidity with which they can purchase business loans in the cash market.

Let $\overline{\nu s}$ denote the system-wide quantity of safe assets sold to the central bank in period 1; in equilibrium, $\overline{\nu s} = \nu s$. The central bank therefore issues $p\overline{\nu s}$ units of reserves, each

[2] Safe assets could be made more liquid by allowing banks to preposition a fraction of their safe-asset holdings in period 1, thereby giving them access to central bank reserves in period 2. We abstract from this possibility.

promising one unit of the consumption good in period 3, and receives $\overline{\nu s}$ units of safe assets, each also returning one unit in period 3. Its profit from the transaction is therefore

$$\Pi^{\text{cb}} = \overline{\nu s} - p\overline{\nu s},$$

so that the central bank breaks even, or earns a surplus, if and only if

$$\Pi^{\text{cb}} \geq 0 \quad \Leftrightarrow \quad p \leq 1.$$

The central bank's profit is assumed to be rebated lump-sum to the households in period 3. As they choose their ν and s in period 1, individual banks do not internalize that the central bank's profit will be rebated. For banks, the relevant comparison is between holding safe assets and holding reserves. Ignoring payments to depositors, an individual bank's expected payoff is

$$W(s,\nu) = (1-s)(1+\rho-\kappa) + s + (p-1)\Big[\nu s - \overline{\nu s}\Big] - \Bigg[\underbrace{\mu(1-s) - (1-\mu)p\nu s}_{\substack{\text{business loans remaining in} \\ \text{bad banks in a crisis}}}\Bigg]\Psi(\hat{R})$$

The first term is the payoff from business loans if all loans were held by good banks, while the second is the payoff from safe assets held in the system. The third term captures the net gain from exchanging one unit of safe assets for p units of reserves. It also reflects an externality: the bank does not internalize that central bank profits are rebated to households, and therefore takes $\overline{\nu s}$ as given. The last term captures the bank's expected cost from the misallocation of business loans in crises.

To see this, suppose first that the bank turns out to be good, which occurs with probability $1-\mu$. In a crisis, it can purchase business loans from bad banks only in the cash market. Since the bank holds $p\nu s$ units of central bank reserves and the crisis cash-market price is $q = 1$, it can purchase $p\nu s$ loans. Aggregating over good banks and crisis states gives the term $(1-\mu)p\nu s\Psi(\hat{R})$. Suppose instead that the bank turns out to be bad, which occurs with probability μ. In a crisis, its own business loans generate no excess return, giving the term $-\mu(1-s)\Psi(\hat{R})$. Aggregating across banks, the quantity of business loans that remains with bad banks in a crisis, and is therefore not efficiently monitored, is

$$\mu(1-s) - (1-\mu)p\nu s.$$

It follows that the bank sells all its safe assets to the central bank whenever

$$\frac{\partial W(s,\nu)}{\partial \nu} > 0 \quad \Leftrightarrow \quad p > \underline{p} \equiv \frac{1}{1 + (1-\mu)\Psi(\hat{R})} \leq 1.$$

Since $\Psi(\hat{R}) \geq 0$, we have $\underline{p} \leq 1$. If $\Psi(\hat{R}) \neq 0$, there exists a range of prices $p \in (\underline{p}, 1]$ such that banks strictly prefer to hold reserves rather than safe assets, while the central bank makes a positive profit on the transaction. The gap between the safe asset's fundamental value, normalized to one, and banks' reservation price, $\underline{p}$, reflects the shadow value of central bank reserves as the only settlement instrument in the interbank cash market. Let $\mathscr{V}$ denote this shadow value, or the "convenience yield" on reserves (López-Salido and Vissing-Jorgensen (2023)). Then

$$\mathscr{V} \equiv 1 - \underline{p} = \frac{(1-\mu)\Psi(\hat{R})}{1+(1-\mu)\Psi(\hat{R})}.$$

Thus, $\mathscr{V}$ increases with the expected cost of a crisis, $\Psi(\hat{R})$, which in turn depends on the policies deployed to mitigate that cost. Absent financial friction, $\Psi(\hat{R}) = 0$, there is no crisis risk and no role for settlement liquidity; in that case, the shadow value of reserves is zero: $\mathscr{V} = 0$. In the presence of financial frictions, $\Psi(\hat{R}) \geq 0$ and $\mathscr{V} \geq 0$ and, in period 1, banks weakly prefer to sell their safe assets to the central bank and hold reserves instead. We can thus set $\nu^{\star} = 1$ without loss of generality, as resulting from a portfolio optimization problem.

C Liquidity Buffers: Usability *versus* Usefulness

This section shows that the dual secondary-market structure of the baseline model is allocation-equivalent to a single secondary market in which reserves remain on good banks' balance sheets. In the baseline model, business loans can be reallocated through two markets: a cash market, in which good banks use reserves to purchase loans, and a credit market, in which borrowing finances additional purchases. In the alternative economy considered here, only the credit market is available. Without a cash market, the good banks cannot draw down their reserves to purchase business loans at date 2. Instead, they keep reserves on their balance sheets, where they serve as "skin in the game" and allow good banks to purchase more business loans on credit.

We show that good banks can in fact purchase on credit as many business loans as they would purchase in total if both the cash and credit markets were available. This equivalence implies that liquidity buffers need not be drawn down —and therefore need not be *usable*— in order to provide liquidity —and hence to be *useful*. By ensuring that reserves remain available to creditors in the event of borrower default, liquidity regulation can support credit-market trade without requiring banks to draw down their buffers in a crisis.[3]

[3] A recurring concern with Basel III liquidity buffers is that banks may be reluctant to draw them down in stress, for fear of market stigma, rating pressure, or uncertainty about how quickly the buffers must be rebuilt. Buffers that are formally usable may therefore be only imperfectly usable in practice — the "last taxi at the taxi stand" problem. Our equivalence result suggests, however, that liquidity buffers can provide liquidity even when they are not drawn down.

To establish this equivalence, we abstract from the cash market described in the main text. The credit market is therefore the only venue in which banks can trade business loans in period 2. Liquidity regulation prevents borrowers from contractually pledging their liquid assets —here, central bank reserves. Reserves must remain unencumbered.

We assume that reserves can be seized costlessly by creditors in period 3 in the event of default: banks cannot abscond with reserves.[4] One interpretation is that liquid assets held to satisfy minimum regulatory requirements are monitored by supervisors. In this sense, banking supervision acts as a third-party trustee, ensuring that a bank's minimum liquidity buffer remains available to —and therefore seizable by— creditors if the bank defaults.

Under this assumption, the incentive-compatibility constraint differs from (IC) and becomes

$$\underbrace{1-s+\ell^{\text{cred}}(s,g;R,r,q)}_{\substack{\text{mimics a good bank,}\\ \text{purchases business loans on credit}\\ \text{and defaults}}} \leq \underbrace{(1-s)(1+r)+s}_{\text{sells business loans on credit}}$$

If a bank borrows $\ell^{\text{cred}}(s,g;R,r,q)$ and absconds, it can abscond only with its portfolio of business loans, $1-s+\ell^{\text{cred}}(s,g;R,r,q)$, because its reserves s are seized by creditors. Reserves thus play a skin-in-the-game role: a bad bank must forgo them if it borrows and absconds. We obtain the borrowing limit:

$$\ell^{\text{cred}}(s,g;R,r,q) \leq \bar{\ell}(s;r) \equiv r(1-s)+s.$$

At the aggregate level, demand for business loans in the credit market is

$$L_D^{\text{cred}}(s;r) \equiv (1-\mu)\ell^{\text{cred}}(s,g;R,r,q) \begin{cases} = -(1-\mu)(1-s) & \text{if } r > R \\ \in \Big[-(1-\mu)(1-s), (1-\mu)\Big[r(1-s)+s\Big]\Big) & \text{if } r = R \\ = (1-\mu)\Big[r(1-s)+s\Big] & \text{if } r < R \end{cases}$$

Supply by bad banks is

$$L_S^{\text{cred}}(s;r) \equiv -\mu\ell^{\text{cred}}(s,b;R,r,q) \begin{cases} = \max\{0\ ;\ \mu(1-s)\} & \text{if } r > 0 \\ \in [0, \max\{0\ ;\ \mu(1-s)\ \}] & \text{if } r = 0 \end{cases}$$

The credit market clears when

$$L_D^{\text{cred}}(s;r) = L_S^{\text{cred}}(s;r).$$

[4]A bank is required to maintain its minimum liquidity buffer at all times while operating as a going concern. This buffer may only be utilized in the event of the bank's default and liquidation —at which point the minimum liquidity requirement ceases to apply.

It freezes when supply exceeds incentive-compatible demand for all feasible credit-market rates $r \in (0, R]$:

$$L_D^{\text{cred}}(s; r) < L_S^{\text{cred}}(s; r) \quad \forall r \in (0, R].$$

Equivalently,

$$(1-\mu)\Big(r(1-s)+s\Big) < \mu(1-s) \quad \forall r \in (0, R].$$

Because demand is increasing in r, this condition holds if and only if it holds at $r = R$, that is,

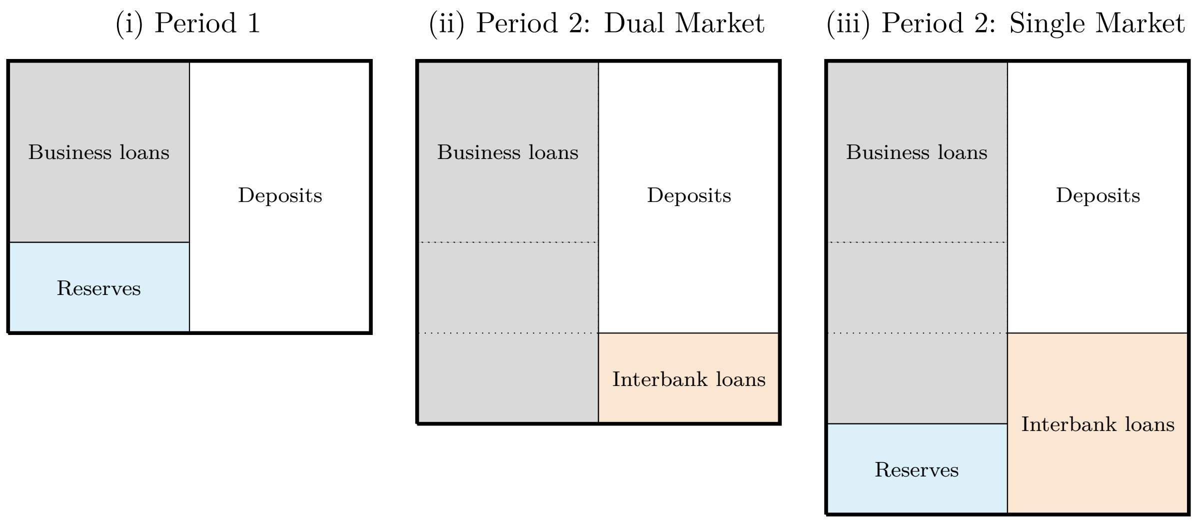
$$R < \hat{R}(s) \equiv \frac{\mu - s}{(1-\mu)(1-s)},$$

which is exactly relation (21). The probability of a crisis is therefore the same as in the dual-market economy. During a crisis, however, the credit market does not fully collapse, because creditors can seize borrowers' reserves. Thus, even when $r = 0$, good banks can still borrow against their reserves:

$$L_D^{\text{cred}}(s; 0) = (1-\mu)s,$$

which corresponds to the amount of business loans that good banks purchase in the cash market during a crisis in the baseline model; recall that $L_D^{\text{cash}}(s; 1) = (1-\mu)s$. We conclude that the dual-market structure with cash and credit markets is allocation-equivalent to the single-market structure with seizable reserves.

Figure C.1: Good Bank's Balance Sheet



Notes: Panel (ii): the good bank purchases business loans on the cash market, in effect swapping its initial reserves for business loans, and borrows on the credit market to finance the purchases of additional business loans. Panel (iii): the good bank keeps its reserves on balance sheet and borrows on the credit market to finance its purchases of business loans. In that scenario, reserves serve as skin-in-the-game, allowing the good bank to borrow a larger amount on the interbank loan market.

Importantly, in the case of the single secondary market, central bank reserves remain on good banks' balance sheets in period 2: as Figure C.1 illustrates, good banks do not draw down their reserves to purchase business loans (compare panels (ii) and (iii)). Reserves are nevertheless useful for financial stability: by serving as skin-in-the-game, they deter default by potential borrowers and relax good banks' borrowing constraint —interbank loans are larger in panel (iii) than in panel (ii).

D Credit-Market Equilibrium Robustness: Individual Price Deviations

The aim of this section is to determine whether equilibria $\mathscr{E}$ and $\mathscr{A}$ are robust to price deviations by individual banks.[5] To fix ideas, we focus on the laissez-faire case.

D.1 Price Deviations from $\mathscr{E}$ Are Not Possible

In equilibrium $\mathscr{E}$, the credit-market rate is $r = R$. A borrower cannot profitably offer a rate above R, since doing so would generate losses; nor can it offer a lower rate, since such an offer would fail to attract lenders. Similarly, a lender cannot profitably offer a rate below R, since this would reduce its payoff; nor can it offer a higher rate, since such an offer would fail to attract borrowers. Hence, whenever equilibrium $\mathscr{E}$ exists, *i.e.* for $R \geq \hat{R}^{\star}$, it is robust to individual price deviations.

D.2 Price Deviations from $\mathscr{A}$

In equilibrium $\mathscr{A}$, the credit-market rate is $r = 0$. Consider an individual lender who deviates by offering to lend at rate

$$\tilde{r} = R - \varepsilon,$$

with $\varepsilon \in (0, R)$, where R is the return on business loans in equilibrium $\mathscr{A}$. As ε approaches zero, $\tilde{r}$ approaches the highest rate a good bank can accept: any rate above R would leave it with no strictly positive surplus from the trade.

We distinguish two cases, depending on whether R is below or above the crisis threshold $\hat{R}^{\star}$, given by (30).

D.2.1 Deviations from $\mathscr{A}$ Are Not Possible When $R < \hat{R}^{\star}$

When $R < \hat{R}^{\star}$, equilibrium $\mathscr{E}$ does not exist, and $\mathscr{A}$ is the unique equilibrium. A lender benefits from lending at rate $\tilde{r}$ only if its expected payoff exceeds the payoff from keeping business loans on its balance sheet, whose gross return is one. Since borrower types are

[5] When individual banks find it optimal to adopt the equilibrium credit-market rate, it is also optimal for them to treat that rate as given, consistent with Definition 1 of a competitive equilibrium.

not observed, the lender is repaid with probability $1-\mu$ and faces default with probability μ. A prospective lender is therefore willing to lend if and only if

$$\underbrace{(1-\mu)(1+R-\varepsilon)+\mu\times 0}_{\text{repaid loans}} > 1. \tag{61}$$

A necessary condition for this inequality to hold for some $R \in [0, \hat{R}^{\star})$ and some $\varepsilon > 0$ is

$$\hat{R}^{\star} > \frac{\mu}{1-\mu},$$

which contradicts Assumption 1, given (30). Hence, equilibrium $\mathscr{A}$ is robust to individual price deviations whenever it is the unique equilibrium.

D.2.2 Deviations from $\mathscr{A}$ Are Possible When $R \geq \hat{R}^{\star}$

When $R \geq \hat{R}^{\star}$, equilibrium $\mathscr{A}$ coexists with equilibrium $\mathscr{E}$. From condition (61), an individual price deviation from $\mathscr{A}$ can succeed whenever

$$R > \frac{\mu}{1-\mu}.$$

Thus, when $\mathscr{A}$ coexists with $\mathscr{E}$, it is not robust to individual price deviations in all states of nature.

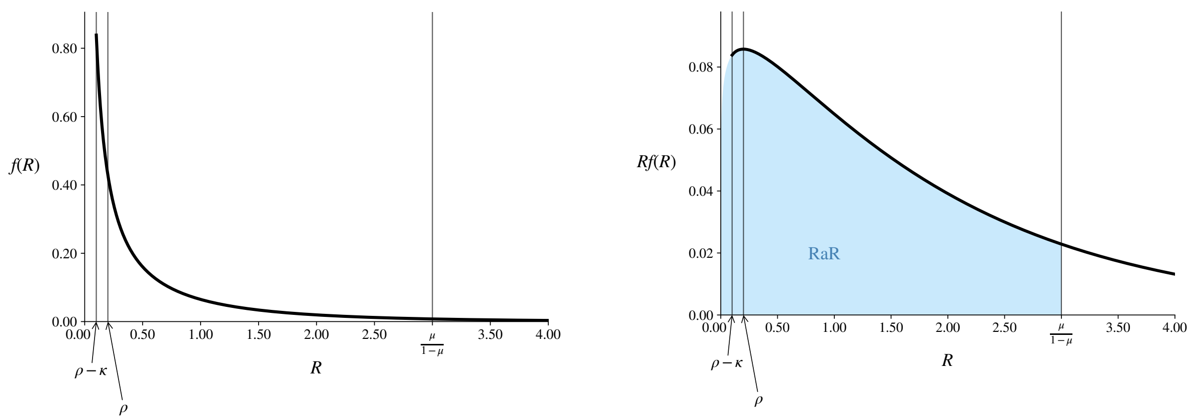
We conclude that equilibrium $\mathscr{E}$ is robust to individual price deviations whenever it exists. Equilibrium $\mathscr{A}$ is robust to such deviations only when it is the unique equilibrium.

E Gamma Distribution

Figure E.1 plots the Gamma density function $f(R)$ and the function $Rf(R)$, using the baseline parameter values of the model. In panel (ii), the shaded area under the curve represents return-at-risk: the expected cost of a crisis under laissez-faire when banks hold no reserves.

Figure E.1: Gamma Distribution

(i) Function $f(R)$ (ii) Function $Rf(R)$ and Return-at-Risk

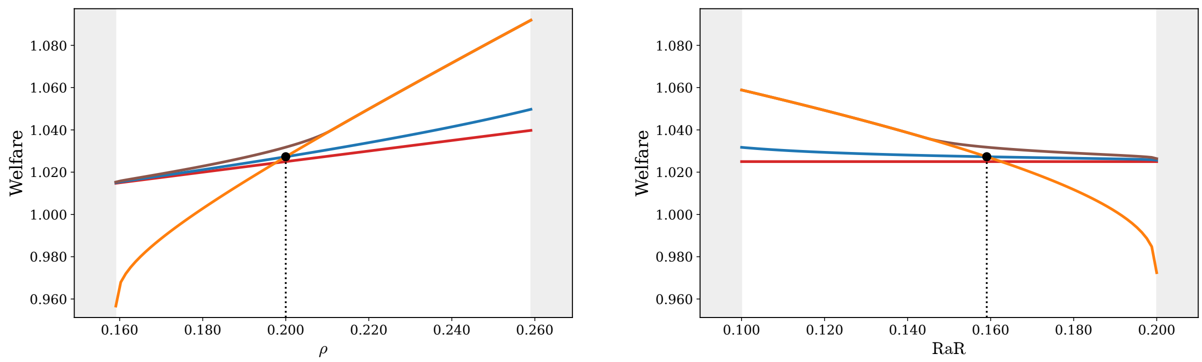


F Policies, Fundamentals and Welfare

Figure F.1 describes how welfare under the various policies varies with the underlying economic fundamentals. In the model, these fundamentals are summarized by two objects: the expected return on business loans, ρ, and the return-at-risk, RaR, which captures the expected cost of a credit-market freeze, or "left-tail risk".

Figure F.1: Welfare Outcomes Vary with Economic Fundamentals

(i) *Change in Expected Returns,* ρ (ii) *Change in Return-at-Risk,* RaR



Optimal liquidity regulation, s_r Backstop, s_b Laissez-faire, $s^\star$ Optimal policy mix, s_m

Notes: Welfare computed for $\kappa = 0.1$ and $\mu = 0.75$. The parameters of the Gamma distribution, k and θ, are set so that RaR ≈ 0.16 and $\rho \in [\text{RaR}, \text{RaR} + \kappa]$ in panel 6i, and so that $\rho = 0.2$ and RaR $\in [\rho - \kappa, \rho]$ in panel 6ii. The black dot corresponds to the baseline as in Figure 5. The gray area is the set of parameter values that are either infeasible (RaR $\geq \rho$) or that violate Assumption 1 (RaR $< \rho - \kappa$).

G Repeated Central Banks' Market Backstops

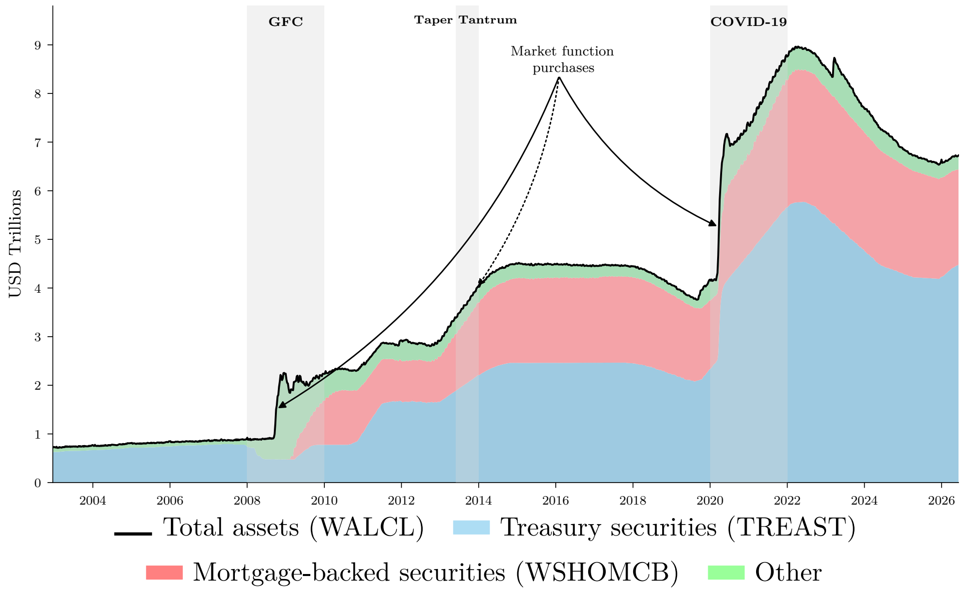
Figure G.1 shows the evolution of the Federal Reserve's balance sheet since early 2002. The figure illustrates a ratchet effect in the level of reserves: each major episode of market stress is followed by a sharp upward shift in reserves, while subsequent normalization remains only partial, possibly reflecting changes in market structure as well as quantitative easing. During the Global Financial Crisis, for example, reserves rose from a near-negligible level to more than USD 1 trillion as the Federal Reserve expanded its balance sheet to stabilize banks and wholesale funding markets. Reserves later declined during tapering and balance-sheet runoff, but did not return to their pre-crisis regime. The COVID-19 shock generated an even larger increase, with reserves rising above USD 4 trillion amid large-scale asset purchases and emergency liquidity support.

The close association between episodes of market stress and actual asset purchases suggests that announcements of future purchases alone —through the so-called "signaling" and "stock" effects— may be insufficient to resolve market freezes.[6] During periods of

[6]The stock effect of central bank asset purchases is also referred to as the "portfolio-rebalancing" effect.

severe market strain, central banks typically rely on *rapid and large-scale* purchases to stabilize asset prices and restore market functioning, a phenomenon known as the "flow effect". Figure G.1 illustrates that market-function purchases are generally larger and more concentrated in time than quantitative-easing purchases. As D'Amico et al. (2026) document, most of the Federal Reserve's asset purchases between March 2020 and May 2022 occurred during the "dash-for-cash" episode from March to June 2020. At that point, the Federal Reserve's primary objective was to restore market functioning —rather than boost the economy, and its purchases were both substantial —conducted "in the amounts needed"— and frequent, often daily or weekly. Asset purchases later shifted toward quantitative easing, aimed at stimulating the broader economy. These later purchases were much smaller in scale, more gradual, and more predictable.

Figure G.1: Central Bank Balance Sheet Size and Composition



Notes: Total assets of the Federal Reserve Banks. The Federal Reserve's large-scale asset purchases during the Global Financial Crisis and the COVID-19 pandemic are prominent examples of market-functioning asset purchases (Fleming et al. (2021); Duffie and Keane (2023)). Strictly speaking, the May–September 2013 Taper Tantrum did not trigger market-function purchases by the Federal Reserve. Following Federal Reserve communications about a prospective reduction in the pace of asset purchases, long-term Treasury and corporate bond yields rose sharply and market liquidity deteriorated, but not nearly to the extent observed during the 2008 crisis or in March 2020. In September 2013, amid tighter financial conditions, the Federal Reserve unexpectedly maintained purchases at their existing monthly pace and postponed the anticipated taper by a few months, until January 2014. This decision continued the existing quantitative easing program rather than initiating a distinct market-function purchase program, but was sufficient to calm markets. Source: FRED.

H Extended Policy Mix: Interest On Reserves

We expand the policy framework by allowing the central bank to remunerate reserves in crisis states.[7] Let $\sigma > 0$ denote the interest rate that the central bank decides to pay on its reserves when there is a risk of a credit-market freeze at date 2, and let the subscript M denote the equilibrium under the optimal *extended* policy mix. All else equal, a positive interest rate on reserves raises the value of central bank reserves relative to business loans, thereby lowering the cash-market price of business loans:

$$q = \frac{1+r}{1+\sigma}$$

The positive interest rate on reserves, and the resulting lower cash price of business loans, changes banks' payoffs and incentive-compatibility constraint, which becomes

$$1 - s_M + \frac{s_M}{q} + \ell^{\text{cred}}(s_M, g; R, r, q) \leq (1 - s_M)(1+r) + (1+\sigma)s_M$$

or equivalently

$$\ell^{\text{cred}}(s_M, g; R, r, q) \leq \bar{\ell}(s_M; r; \sigma) \equiv r\left[1 - \frac{r-\sigma}{1+r}s_M\right]. \tag{62}$$

Banks' borrowing capacity $\bar{\ell}(s; r; \sigma)$ is increasing in σ. In a crisis, the central bank can set σ up to R: above that rate, good banks would prefer to retain reserves on their balance sheets rather than use them to purchase business loans in the cash market. It follows that good banks' borrowing limit is maximal, and misallocation is minimal, when the central bank sets the interest rate on reserves at

$$\sigma = R \tag{63}$$

Under the optimal extended policy mix, we therefore have

$$r = \sigma = R$$

which in turn implies

$$q = 1 \tag{64}$$

and

$$\bar{\ell}(s_M; R; R) \equiv R$$

[7] Paying interest on reserves is another example of the unconventional measures that central banks adopted in response to the Global Financial Crisis. In October 2008, for example, the Federal Reserve announced that it would begin to pay interest on depository institutions' required and excess reserve balances. Likewise, the Bank of Japan introduced a complementary deposit facility that paid positive interest on excess reserve balances, with the stated objective of ensuring stability in money markets.

By lowering the cash-market price to $q = 1$, the optimal policy also enables good banks to purchase more business loans in the cash market. In particular,

$$L_D^{\text{cash}}(s_M; q) = (1-\mu)\frac{s_M}{q} = (1-\mu)\frac{1+\sigma}{1+r}s_M = (1-\mu)s_M$$

The residual supply of loans to the credit market is therefore lower than absent interest on reserves:

$$L_S^{\text{cred}}(s_M; r) = \mu(1-s_M) - (1-\mu)s_M = \mu - s_M$$

The extended policy mix gives rise to two crisis thresholds. The first is $\hat{R}_M$, below which the central bank must intervene to backstop the credit market. The second is a lower threshold, $\underline{R}_M < \hat{R}_M$. For $R \in [\underline{R}_M, \hat{R}_M]$, setting the return on reserves at $\sigma = R$ is sufficient to stabilize the credit market, so the central bank need not purchase business loans. For $R < \underline{R}_M$, by contrast, the central bank must supplement interest on reserves with purchases of business loans in the cash market. Even then, the required volume of purchases is smaller than it would be without interest on reserves. A quantitative backstop is required when private demand in the credit market falls short of residual supply:

$$L_D^{\text{cred}}(s_M; R) < L_S^{\text{cred}}(s_M; R)$$

$$\Leftrightarrow (1-\mu)R < \mu - s_M$$

or, equivalently,

$$R < \underline{R}_M \equiv \frac{\mu - s_M}{1-\mu}. \tag{65}$$

From relation (21), it is clear that

$$\underline{R}_M \leq \hat{R}_M$$

When $R < \underline{R}_M$, the central bank must purchase

$$S(R) = L_S^{\text{cred}}(s_M; R) - L_D^{\text{cred}}(s_M; R) = \mu - s_M - (1-\mu)R$$

business loans in the cash market, at the price $q = 1$, in order to restore credit-market functioning. Altogether, purchases of business loans and interest on reserves generate central bank losses:

$$E(\Pi^{\text{cb}}) = -\int_0^{\underline{R}_M} RS(R)f(R)dR - s_M \int_0^{\hat{R}_M} Rf(R)dR$$

Welfare is the sum of banks' expected payoffs from business loans, the payoff on reserves, and central bank profits. Hence,

$$W_M(s_M, \underline{R}_M) = 1 + (1-\mu)(\rho-\kappa) + (\mu - s_M)\left[\rho - \kappa - \Psi(\underline{R}_M)\right] + (1-\mu)\int_0^{\underline{R}_M} R^2 f(R)dR$$

Replacing s_M by its expression as a function of $\underline{R}_M$ in (65), we obtain

$$W_M(\underline{R}_M) = 1+(1-\mu)(\rho-\kappa)+(1-\mu)\underline{R}_M\Big[\rho-\kappa-\Psi(\underline{R}_M)\Big]+(1-\mu)\int_0^{\underline{R}_M} R^2 f(R)dR \quad (66)$$

The derivative with respect to $\underline{R}_M$ has the sign

$$W'_M(\underline{R}_M) \geq 0 \quad \Leftrightarrow \quad \Psi(\underline{R}_M) \leq \rho-\kappa. \quad (67)$$

Since $\Psi'(\underline{R}_M) > 0$ over the feasible range of crisis thresholds, $\underline{R}_M < \mu/(1-\mu)$, condition (67) characterizes an interior maximum when it holds with equality. In that case, the optimality condition coincides with the private first-order condition in (30), which implies

$$\underline{R}_M = \hat{R}^\star$$

and therefore, using (21) and (65),

$$s_M < s^\star$$

Hence, the optimal extended policy mix also entails a lower liquidity requirement than under laissez-faire. Substituting the first-order condition (67) into (66) yields

$$W_M = 1 + (1-\mu)(\rho-\kappa) + \chi_M$$

where

$$\chi_M \equiv (1-\mu)\int_0^{\underline{R}_M} R^2 f(R)dR$$

We can now describe the optimal extended policy mix.

Theorem 10 (Extended Policy Mix). *The central bank can restore market functioning and improve welfare by remunerating reserves in crisis states at date 2, rather than leaving reserves unremunerated. In that case:*

1. *The optimal extended policy mix sets the interest rate on reserves equal to the efficient rate of return on business loans: $\sigma = R$;*

2. *The optimal policy mix follows a pecking order: the central bank first pays an interest rate on reserves and, if this measure proves insufficient to restore market functioning, it proceeds to purchase business loans;*

3. *The market backstop principle described in Theorem 8 continues to hold;*

4. *Welfare is higher under the optimal extended policy mix than under the optimal plain-vanilla policy mix: $W_M > W_m$.*

Proof. Points *1–3* follow from the preceding analysis, so it remains only to prove point *4*. Welfare under the optimal extended policy mix exceeds welfare under the optimal plain-vanilla policy mix if and only if

$$W_M > W_m \Leftrightarrow \chi_M > \chi_m \Leftrightarrow (1-\mu)\int_0^{\underline{R}_M} R^2 f(R)dR > (1-\mu)^2 \int_0^{\hat{R}_m} R^2 f(R)dR$$

$$\Leftrightarrow \int_0^{\hat{R}^\star} R^2 f(R)dR > (1-\mu)\int_0^{\hat{R}_m} R^2 f(R)dR \tag{68}$$

where, from (53)–(54) and (67),

$$\Psi(\hat{R}_m) - \Psi(\hat{R}^\star) = (1-\mu)\int_0^{\hat{R}_m} R^2 f(R)dR.$$

Equivalently,

$$\int_{\hat{R}^\star}^{\hat{R}_m} Rf(R)dR = (1-\mu)\int_0^{\hat{R}_m} R^2 f(R)dR. \tag{69}$$

Since $R \leq \hat{R}_m \leq \frac{\mu}{1-\mu}$, we have

$$\int_{\hat{R}^\star}^{\hat{R}_m} R^2 f(R)dR = \int_{\hat{R}^\star}^{\hat{R}_m} R[Rf(R)]dR < \frac{\mu}{1-\mu}\int_{\hat{R}^\star}^{\hat{R}_m} Rf(R)dR. \tag{70}$$

Decomposing (69), we obtain

$$\begin{aligned}\int_{\hat{R}^\star}^{\hat{R}_m} Rf(R)dR &= (1-\mu)\int_0^{\hat{R}^\star} R^2 f(R)dR + (1-\mu)\int_{\hat{R}^\star}^{\hat{R}_m} R^2 f(R)dR \\ &< (1-\mu)\int_0^{\hat{R}^\star} R^2 f(R)dR + \mu\int_{\hat{R}^\star}^{\hat{R}_m} Rf(R)dR.\end{aligned}$$

Hence,

$$\Leftrightarrow \int_{\hat{R}^\star}^{\hat{R}_m} Rf(R)dR < \int_0^{\hat{R}^\star} R^2 f(R)dR.$$

Finally, applying (69) to the left-hand side gives

$$(1-\mu)\int_0^{\hat{R}_m} R^2 f(R)dR < \int_0^{\hat{R}^\star} R^2 f(R)dR$$

and therefore, from (68), $W_M > W_m$. □

Remunerating reserves in crisis states improves welfare relative to the plain-vanilla policy mix without interest on reserves. The associated minimum liquidity requirement remains modest, *i.e.*, below its laissez-faire level. Moreover, the central bank purchases only the amount of distressed assets needed to restore market functioning, at the fair price. The market backstop principle stated in Theorem 8 therefore continues to hold.