Monetary Policy and Endogenous Financial Crises

F. Boissay* F. Collard† J. Galí‡ C. Manea§¶

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Abstract

What are the channels through which monetary policy affects financial stability? Can (and should) central banks prevent financial crises by deviating from price stability? To what extent may monetary policy itself unintentionally breed financial vulnerabilities? We answer these questions using a New Keynesian model with capital accumulation and endogenous financial crises due to adverse selection and moral hazard in credit markets. Our findings are threefold. First, monetary policy affects the probability of a crisis both in the short run (*via* aggregate demand) and in the medium run (*via* capital accumulation). Second, the central bank can reduce the incidence of crises in the medium run by tolerating higher inflation volatility in the short run. Third, prolonged periods of loose monetary policy followed by a sharp tightening can lead to financial crises.

Keywords: Inflation targeting, low–for–long policy rates, adverse selection, financial crises

JEL classification: E1, E3, E6, G01.

*Bank for International Settlements. Email: frederic.boissay@bis.org.

†Toulouse School of Economics (CNRS). Email: fabrice.collard@gmail.com.

‡CREI, Universitat Pompeu Fabra and Barcelona School of Economics. Email: jgali@crei.cat.

§Bank for International Settlements. Email: cristina.manea@bis.org.

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"*Credit markets are characterized by imperfect and asymmetric information. These informational frictions can interact with other economic forces to produce periods of credit–market stress (...). A high level of credit–market stress, as in a severe financial crisis, may in turn produce a deep and prolonged recession.*"

Bernanke (2023), Nobel Prize Lecture, p. 1

"*A prolonged period of low interest rates can create incentives for agents to take on greater credit risks in an effort to reach for yield.*"

Stein (2013), p. 6

1 Introduction

The impact of monetary policy on financial stability remains a controversial topic. On the one hand, loose monetary policy may stave off a financial crisis by backstopping the financial sector in the face of unexpected adverse circumstances (*e.g.* the Covid–19 pandemic). On the other hand, keeping policy rates low for long can contribute to the build–up of financial vulnerabilities by fueling an excessive credit and asset price boom, as shown by recent empirical studies (e.g. Grimm et al. (2023), Jiménez et al. (forthcoming)).[1]

This ambivalence prompts the question of the appropriate monetary policy in an environment where credit markets are fragile and financial crises may have varied causes. *What are the channels through which monetary policy affects financial stability? Can (and should) central banks prevent financial crises by deviating from price stability? To what extent may monetary policy itself unintentionally breed financial vulnerabilities?*

We study these questions through the lens of a novel New Keynesian (NK) model that features a credit market subject to informational frictions leading to adverse selection and moral hazard. As in Bernanke and Gertler (1990), Azariadis and Smith (1998), and Boissay et al. (2016), adverse selection and moral hazard surface when the real return on capital is low: a low capital return may prompt some borrowers to invest in alternative ("below–the–radar") projects that are privately beneficial but raise the probability of credit default to the detriment of lenders —a behavior sometimes dubbed "search for yield" (Martinez-Miera and Repullo (2017)).[2] In

[1] A case in point is the 2007–08 financial crisis. Taylor (2011) refers to the period 2003–05 in the United States as the "Great Deviation", which he characterizes as a period in which monetary policy became less rule–based, less predictable, and excessively loose. Other notable examples of financial crises preceded by "low–for–long" policy rates include (among many others) the Japanese (Ito and Mishkin (2006)) and Swedish (Englund (1999)) crises in the early 1990s.

[2] In practice, search–for–yield behavior may take various forms, including excessive/reckless risk–taking, false information disclosure, scams, and outright embezzlement (Mishkin (1991), FCIC (2011), Piskorski et al. (2015), Garmaise (2015), Mian and Sufi (2017), Griffin (2021)). FCIC (2011), Garmaise (2015) and Mian and Sufi (2017), for example, provide evidence that subprime borrowers fraudulently overstated their assets and income to obtain loans during the 2002–06 mortgage credit boom in the United States, which eventually culminated in the financial panic of 2007 (Gorton (2008, 2009)).

turn, a low return has varied causes, such as a large adverse non–financial shock or a protracted investment boom fueled by low–for–long monetary policy rates. In the latter case, the longer the period of low policy rates, the longer the boom is likely to last and the larger the capital stock becomes. Because of decreasing marginal returns, the accumulation of capital eventually exhausts profitable investment opportunities and erodes the real return on capital, prompting borrowers to search for yield. The consequent rise in moral hazard and credit default risk may then induce prospective lenders to panic and refuse to lend, triggering a sudden collapse of the credit market, *i.e.* a "financial crisis". Even though default risk is limited to some specific (subprime) borrowers in our model, the adverse selection problem and resulting uncertainty as to where exposures to such borrowers reside suffice to sap lenders' confidence. Hence, the root of financial fragility in our model is not borrowers' defaults *per se* —which are off–equilibrium outcomes— but rather lenders' fear of being defaulted upon.

Our model aligns with narratives of historical financial crises in which fraudulent or reckless behavior by certain borrowers, creditors' limited information regarding borrowers' creditworthiness, and sudden shifts in confidence act as destabilizing forces (see, *e.g.*, Mishkin (1991), Jordà et al. (2017), Baron et al. (2020) and accompanying documentation). For example, at the onset of the Global Financial Crisis (henceforth, GFC), it was the false disclosure of information about subprime borrowers and creditors' concerns about their ultimate exposures to such borrowers (*e.g.* through asset–backed securities) that reportedly led creditors to panic and stop lending —not realized losses *per se* (Gorton (2008), Bernanke (2018)).[3] Our focus on creditors' lack of information and fear of default complements other approaches to introducing financial crises in macroeconomic models. Those approaches emphasize distinct destabilizing forces such as irrational swings in investor sentiment (Bordalo et al. (2018), Fontanier (2025)), sunspots and self–fulfilling bank runs (Gertler and Kiyotaki (2011), Gertler et al. (2020)), and/or financial accelerator mechanisms operating through collateral value, borrowers' net worth, and (realized) defaults (*e.g.* Bernanke and Gertler (1989), Bernanke et al. (1996), Kiyotaki and Moore (1997), Brunnermeier and Sannikov (2014), Christiano et al. (2014)).

To embed informational frictions in a monetary model, we extend the baseline NK model with endogenous capital accumulation in three important ways.

First, we introduce a credit market that reallocates capital among heterogeneous firms. In the spirit of Bernanke and Gertler (1990) and Khan and Thomas (2013), we assume that firms are subject to transitory idiosyncratic productivity shocks —in addition to the usual persistent aggregate ones. This heterogeneity induces productive firms to borrow funds in a credit market to buy capital from unproductive firms, and induces unproductive firms to lend the proceeds

[3] To be sure, our intention is not to claim that borrowers do not default in the wake of financial crises —they do. Rather, we aim to capture the observation that creditors' fear of default plays an important role at the onset of financial crises. In the case of the GFC, Gorton (2010) notes (p. 6): "*Subprime was never large enough to be* [by itself] *an issue for the global banking system. In 2007, the subprime stood at about \$1.2 trillion outstanding, of which roughly 82 percent was rated AAA and to date has very small amounts of realized losses. Yes, \$1.2 trillion is a large number, but for comparison, the total size of the traditional and parallel banking systems is about \$20 trillion*"; Gorton (2008): *"As with the earlier panics, the problem at root is a lack of information"* (p. 2); "*The panic was rooted in fear of losses, the location and extent of which cannot be determined"* (p. 62).

of the sale of their capital goods. Our modeling in terms of inter–firm lending should not be taken literally; rather, it should be understood as broadly capturing the role of the financial system (including financial intermediaries) in reallocating initially mis–allocated resources.[4] A well–functioning credit market helps channel capital from unproductive to productive firms and increases aggregate productivity. By contrast, a dysfunctional credit market induces capital misallocation and a fall in aggregate productivity.

Second, we introduce frictions in this credit market. We assume that borrowers have private information about their productivity and that firms may borrow, engage in below–the–radar activities, abscond, and default. To induce unproductive firms (the natural lenders) to sell their capital stock and lend the proceeds of the sale —rather than borrow and abscond— the equilibrium loan rate must be above a minimum threshold. At the same time, for productive firms (the natural borrowers) to afford a loan, their return on capital must be above the loan rate. It follows that, when the return on capital falls below the minimum loan rate threshold, productive firms cannot (afford to) induce the unproductive ones to lend. Since *all* firms may want to borrow in that case, the lack of information about borrowers and the resulting adverse selection problem kick in, causing prospective lenders to panic and hoard their capital goods.[5]

The third departure from the baseline NK model is that we allow the economy to deviate substantially from its steady state. All else being equal, the credit market is fragile when the capital stock is well above its steady state and the return on capital is low. Consequently, financial crises may occur on the back of a prolonged credit/investment boom (as documented *e.g.* by Schularick and Taylor (2012), Gorton and Ordoñez (2020)). We solve the model globally to capture the non–linearities embedded in the endogenous booms and busts of the credit market.

The baseline version of our model features both aggregate supply and demand shocks and assumes that monetary policy is conducted according to the Taylor (1993) rule. We set the non–financial parameters of the model to conventional values and the financial parameters so that, in the simulated stochastic steady state, the economy spends 10% of the time in a financial crisis and aggregate productivity falls by 1.8% due to financial frictions in a crisis, as observed in OECD countries. Despite its stylized nature, our model does a fair job in capturing and articulating several salient facts about historical financial crises: the average crisis follows an economic and financial boom, a period of low inflation and a U–shaped path of monetary policy rates; it also induces a discrete drop in aggregate productivity and a severe recession.

[4] In the context of a model with heterogeneous firms similar to ours, Bernanke and Gertler (1990) note (p. 94) that "*It is convenient to think of this [inter–firm] borrowing as being channeled through competitive financial intermediaries, which use no resources in the process of intermediation and earn no profits in equilibrium*". In Section L.1 of the Online Appendix, we present a version of our model with banks, in which productive firms borrow from banks to buy capital goods from unproductive firms and the latter deposit the proceeds of the sale in the banks. Abstracting from nominal rigidities, our model is a simplified version of the model in Boissay et al. (2016), but with heterogeneous firms instead of heterogeneous banks.

[5] Our view of financial crises is akin to Dang et al. (2020)'s information view. This view holds that when economic fundamentals are strong, lenders are confident that borrowers will repay their loans and therefore lend "without asking questions" —lending is information–insensitive. When fundamentals are weak, in contrast, lenders may worry that some borrowers will default and, without information about prospective borrowers, may refuse to lend —lending becomes information–sensitive.

To study the effects of monetary policy on financial stability, we compare the dynamics and welfare performance of our model economy under different Taylor–type rules as well as regime–contingent monetary policy rules. We also consider the effects of random deviations from the monetary policy rule.

Our main findings are threefold.

First, the central bank may affect the probability of a crisis both in the short run and over the medium run through different channels. In the short run, it may do so through the effects of contemporaneous changes in its policy rate on output and inflation. For example, a rate hike reduces aggregate demand and weighs on productive firms' return on capital, which brings the economy closer to a crisis. In the medium run, monetary policy affects financial stability through its impact on households' saving behavior and capital accumulation. For example, a central bank that commits itself to systematically and forcefully responding to fluctuations in output will —all else equal— tend to slow down capital accumulation during investment booms, thus enhancing credit market resilience.

Second, the central bank faces a trade–off between price stability and financial stability. We show that a central bank can significantly reduce the incidence of financial crises in the medium term by tolerating higher inflation volatility in the short run (*e.g.*, by responding systematically more to fluctuations in output, all else equal). We also show that one way to alleviate the trade–off between price stability and financial stability is to follow a more flexible (regime–contingent) policy rule, whereby the central bank commits to price stability in normal times *and* to doing whatever is needed to forestall a crisis in times of financial stress. Such a "backstop" policy entails exceptionally loose monetary policy in times of financial stress and yields significant welfare gains relative to *strict* inflation targeting (henceforth, SIT).[6]

Third, we emphasize the opposite effects on financial stability of cutting the policy rate *versus* keeping it low for long. On the one hand, and all else equal, unexpected rate cuts boost aggregate demand and raise the return on capital, which dissuades investors from searching for yield. Hence, a temporary rate cut helps lower the probability of a crisis in the short run while, on the flip side, a hike may trigger a crisis. On the other hand, keeping the policy rate low for a long time stimulates the accumulation of capital and gradually erodes the return on capital, eventually prompting investors to search for yield. The upshot is that a crisis is more likely when the central bank unexpectedly tightens monetary policy after having kept it loose for long, consistent with the empirical evidence from Schularick et al. (2021), Grimm et al. (2023) and Jiménez et al. (forthcoming).

Relation and Contribution to the Literature. Our paper is related to several strands of the literature. Perhaps the closest one features macroeconomic models that emphasize the role of informational frictions in credit markets. Our model is notably reminiscent of Bernanke and

[6] One novel feature of our model is that it accounts for the role of monetary policy not only as a tool to achieve price stability but also as a potential tool to restore credit markets' functionality (*e.g.* Bank for International Settlements (2022), Duffie and Keane (2023)).

Gertler (1990), Gertler and Rogoff (1990), Azariadis and Smith (1998) and Boissay et al. (2016), in which borrowers differ in terms of their riskiness/productivity and have an informational advantage over lenders that gives rise to moral hazard and sudden credit market shutdowns —*i.e.* "financial crises".[7] This "information view" of financial crises can be motivated by two complementary sets of historical studies: those that emphasize the panic–like aspect of past crises and ascribe panics to lenders' lack of information about borrowers, such as Gorton (1988), Mishkin (1991), Gorton (2009), Bernanke (2018); and those that explain the presence of moral hazard by a deterioration of macroeconomic fundamentals, possibly due to loose–for–long monetary policy, such as Corsetti et al. (1999) and Jiménez et al. (2014).[8] Our contribution is to embed this view of financial crises into an otherwise standard dynamic stochastic general equilibrium NK framework with fully rational expectations in which there is a role for monetary policy.

Following Kiyotaki and Moore (1997), Gertler and Kiyotaki (2011), Gertler and Karadi (2011), Jermann and Quadrini (2012), a large body of the macro–financial literature models crises as situations where borrowers' financing constraint (*e.g.* a leverage or collateral constraint) tightens in the wake of an exogenous adverse financial shock (*e.g.* capital quality shock, capital pledgeability shock or risk shock). In more recent papers (*e.g.* Gertler and Kiyotaki (2015), Boissay et al. (2016), Gertler et al. (2020)) financial crises take the form of endogenous panics, as in our case. Our paper complements this previous work by focusing on the role of monetary policy in the genesis of financial crises.

We are not the first to model the effects of credit market disruptions on factor misallocation and aggregate productivity (*e.g.* Khan and Thomas (2013), Moll (2014), Midrigan and Xu (2014), Ottonello (forthcoming)).[9] One common feature of existing contributions is that credit disruptions take the form of large exogenous adverse financial shocks. In our case, in contrast, financial crises are triggered by standard adverse non–financial shocks on the back of endogenous financial imbalances. Our approach can therefore be seen as endogenizing the financial shocks typically considered in the macro–financial literature (Galí (2018)).

The paper proceeds as follows. Section 2 reviews key stylized facts of financial crises. Sections 3 and 4 describe our theoretical framework and the channels through which monetary policy affects financial stability. Section 5 presents the dynamics around financial crises in the model.

[7] In this class of models, as in other classical principal–agent models of the credit market (*e.g.* Stiglitz and Weiss (1981), Mankiw (1986)), the aggregate outcome improves with the "creditworthiness" of borrowers, reflected in their net worth, payoffs, or capital returns. One difference, however, is that the loan market rate affects incentives in opposite ways. In Stiglitz and Weiss (1981), for example, a rise in the loan rate along the credit supply curve crowds out the *safest* borrowers, whereas in Bernanke and Gertler (1990), Azariadis and Smith (1998) and our model, it instead crowds out the *riskiest* borrowers. The difference is due to the fact that, in the latter models, firms also hold internal funds and have the option to lend these funds in the credit market. When inefficient firms have internal funds, a rise in the loan rate *increases* their opportunity cost of investing in risky projects.

[8] Related contributions include Mishkin (1999), Brunnermeier (2009), Gorton and Metrick (2012), who also consider the combination of asymmetric information and moral hazard as the root cause of financial panics; and Stein (2013), Grimm et al. (2023) and Jiménez et al. (forthcoming), who also highlight the link between low–for–long policy rates and excessive risk–taking/moral hazard.

[9] There is also extensive empirical evidence that financial crises are associated with capital misallocation and a fall in aggregate productivity (Gopinath et al. (2017), Foster et al. (2016), Duval et al. (2019)).

Section 6 revisits the "divine coincidence" result and studies whether a central bank should depart from its objective of price stability to prevent financial crises. Section 7 studies the effect of monetary policy surprises on financial stability and shows how random deviations from the monetary policy rule can breed financial vulnerabilities. A final section concludes.

2 Salient Facts about Financial Crises

Our model is meant to be consistent with salient facts about financial crises that are common across a broad range of historical episodes. It is not meant to provide a detailed description of one particular crisis.

The extant empirical literature has identified several salient facts, which we recapitulate and illustrate in Figure 1. In Section 5 we will return to these facts and articulate them together in light of our model.

Figure 1 reports the dynamics of key macro–financial variables around post–WW2 financial crises in 18 advanced economies. It focuses on nine variables relating to output, credit, capital, productivity, capital returns, inflation, and monetary policy. Most of the data are from the latest release of the Jordà–Schularick–Taylor Macrohistory Database (Jordà et al. (2017) and Jordà et al. (2019), henceforth JST). The only exceptions are asset prices —which we take from Global Financial Data (like Greenwood et al. (2022)); the capital stock and the output–to–capital ratio —from IMF (2021)'s Capital Stock Dataset (see also Gupta et al. (2014)); and total factor productivity adjusted for labor utilization —from Jordà et al. (2024).[10] The list of financial crises and their starting years are taken from JST as well. In line with other databases (*e.g.* Laeven and Valencia (2018), Baron et al. (2020)), a financial crisis in JST is defined as "*instances of major bank failures, banking panics, substantial losses in the banking sector, significant recapitalization, and/or significant government intervention.*" (JST documentation, p. 1). A cross–check against the Baron et al. (2020) database indicates that the bulk (22 out of 25) of the post–WW2 crises in JST feature a financial panic —defined as "*an episode of severe and sudden withdrawals of funding by (...) creditors from a significant part of the (...) system* (Baron et al. (2020), p. 53)", which is the type of crisis considered in our model.

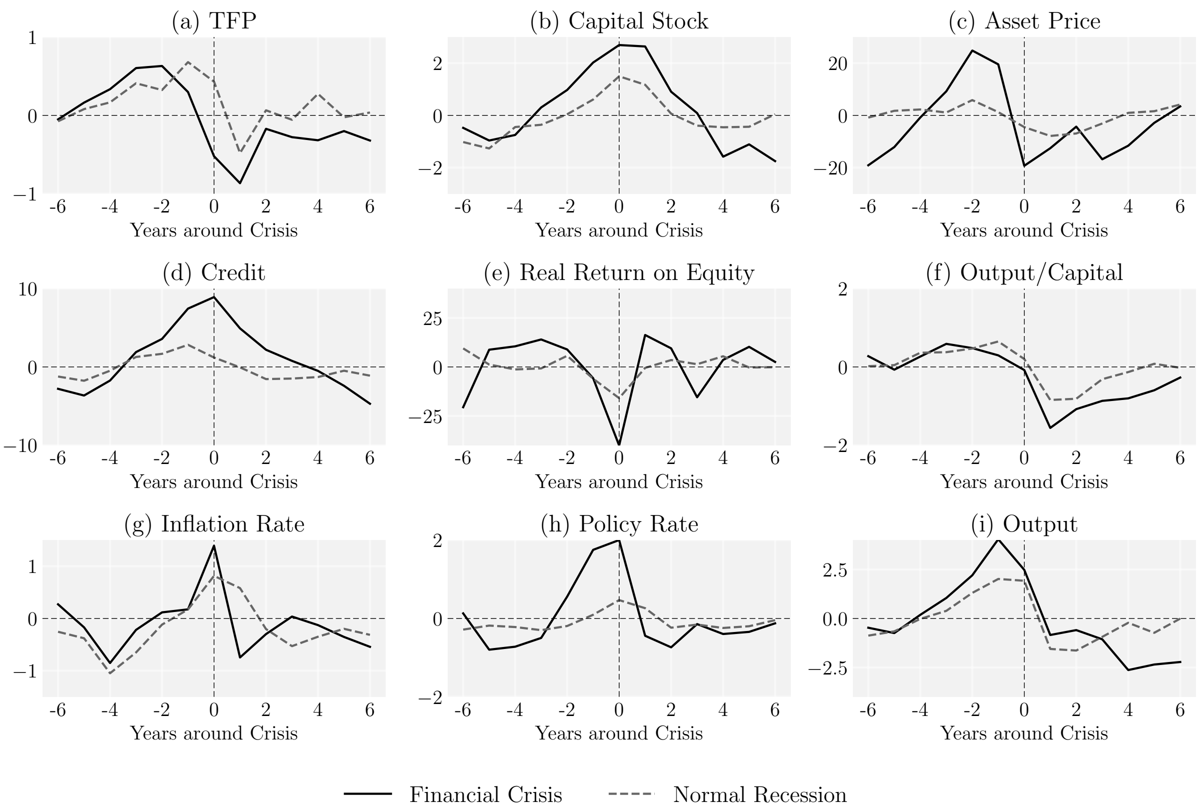
Crises Cause Severe Recessions. One of the most salient facts about financial crises is that they tend to cause relatively deep and long–lasting recessions (Cerra and Saxena (2008) and Jordà et al. (2013); panel (i), solid *versus* dashed line). Recent studies explain the severity of such "financial recessions" by their coincidence with unusually large and prolonged falls in total factor productivity compared to normal recessions (Oulton and Sebastiá-Barriel (2016), Ikeda and Kurozumi (2019)). In turn, the fall in aggregate productivity during crises has been shown to be due (at least in part) to firms' financing constraints, which tend to hinder the efficient

[10] For a detailed description of the data used in Figure 1, see Section A of the appendix. We thank Òscar Jordà, Sanjay Singh, and Alan Taylor for sharing their utilization–adjusted productivity data.

allocation of capital across businesses (Foster et al. (2016), Gopinath et al. (2017), Duval et al. (2019); panel (a), solid *versus* dashed line).

Crises Follow Credit/Investment and Asset Price Booms. Another well–documented fact is that financial crises tend to follow unusually large booms in credit and asset prices (*e.g.* Borio and Lowe (2002), Schularick and Taylor (2012); Figure 1, panels (c) and (d), solid *versus* dashed line). Greenwood et al. (2022) document that a significant proportion (between 40% and 64%) of financial crises follow such financial booms. The financial booms that precede crises also tend to go hand in hand with investment booms (see, *e.g.* Gorton and Ordoñez (2020) or Gorton and Ordoñez (2023), chapter 1) and with sustained capital accumulation (panel (b)).

Figure 1: Median Dynamics Around Financial Crises



Notes: Median dynamics of key macro–financial variables around financial crises in the post–WW2 period. All variables are annual and de–trended using Hodrick and Prescott (1997) with $\lambda = 100$. Vertical line (year = 0): first year of the crisis/recession. The starting years of financial crises and recessions are taken from Jordà et al. (2017) and Jordà et al. (2013), respectively. Except for the inflation rate (panel (g)), one obtains similar dynamics when the sample period is expanded back to 1870. For the dynamics over the full sample period (1870–2020), see Section A of the Online Appendix. For all variables, the *median* crisis dynamics are essentially the same as the *average* ones. We report the median only to emphasize that the results are not driven by specific or extreme crisis episodes.

That said, not all booms lead to crises. Dell'Ariccia et al. (2016) and Greenwood et al. (2022) document that, on average, "only" one in three booms is followed by a financial crisis. Recent empirical studies have in turn shown that such "bad booms" are associated with specific dynamics of aggregate productivity and monetary policy rates.

Crises Follow Productivity Slowdowns. Bad and good booms differ in terms of their underlying productivity dynamics. Gorton and Ordoñez (2020, 2023) show that, while both types of booms are initially caused by a rise in aggregate productivity, the productivity gains "die off" faster in bad booms than in good booms. Relatedly, Paul (2023) finds that a decline in aggregate productivity is a robust predictor of financial crises at a two–year horizon. These findings are illustrated in Figure 1 (panel (a), solid line), which shows that productivity begins to decline as early as two years before the median crisis and continues to fall in its aftermath. These findings also align well with the narratives of several past crisis episodes. For example, the Japanese banking crisis in the early 1990s followed the boom–bust of the Japanese electronics sector (Hayashi and Prescott (2002), Cao and L'Huillier (2018)), while the GFC followed the boom–bust in information technologies of the early 2000s (Fernald (2015)).

Crises Follow a U–shaped Monetary Policy. Another characteristic of bad booms is their association with a U–shaped path of monetary policy rates —defined as a prolonged period of relatively low rates followed by rapid hikes (Figure 1, panel (h)). Grimm et al. (2023) provide empirical evidence that discretionary decisions to keep monetary policy loose for an extended period of time can cause a boom in credit and beget financial vulnerabilities down the road. In parallel, Schularick et al. (2021) show that discretionary hikes in the policy rate during a credit boom may trigger a financial crisis. Together, these findings align with those of Jiménez et al. (forthcoming), who establish a transmission chain that links U–shaped monetary policy rates, search–for–yield behavior and the subsequent financial crisis. At first, rate cuts boost the supply of credit and aggregate investment. But the longer the period of low rates, the scarcer the profitable lending opportunities (panels (e) and (f)), and the more likely it is that credit flows toward riskier or less productive investments, stoking financial vulnerabilities. When the central bank eventually hikes its policy rate, these vulnerabilities come to the fore and a crisis breaks out. In addition, a few empirical works have linked the U–shaped monetary policy observed during bad booms to the coincident U–shaped dynamics of inflation (panel (g)). Bordo and Wheelock (2007) and Ikeda (2022), for example, document that asset price booms tend to arise during periods of above–average growth of real output, below–average inflation and low policy rates, and end within a few months of an increase in inflation and monetary policy tightening.[11]

3 Model

Our model is a variant of the textbook NK model (Galí (2015)), with sticky prices *à la* Rotemberg (1982), capital accumulation, and financial frictions.

[11] One well–known example of a crisis that broke out after a protracted disinflationary boom and as the central bank raised its policy rate is the 1990–91 Japanese banking crisis. Okina et al. (2001) report that in 1987 the Bank of Japan considered raising its policy rate to rein in the asset price boom but, with inflation close to 0%, lacked the arguments to justify a tightening and eventually raised its policy rate only in 1989, after inflation rose above 2%. More generally, Borio (2006) notes that (p. 5) "*Financial imbalances can and do also develop in a low inflation environment. (...) And low inflation, by obviating the need to tighten monetary policy, can also remove a key constraint on the development of the imbalances*".

The model economy is populated by a central bank, a continuum of identical households, a continuum of differentiated and monopolistically competitive retailers indexed by $i \in [0,1]$, as well as a unit–sized continuum of competitive intermediate goods producers —henceforth "firms". Firms are the only non–standard agents in the model: they are subject to idiosyncratic productivity shocks, which induce them to rescale their capital stock and participate in a credit market.

3.1 Central Bank

The central bank sets the policy rate i_t according to the following Taylor–type rule:[12]

$$1 + i_t = \frac{1}{\beta}(1+\pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \tag{1}$$

where $1/\beta$ is the gross natural rate of interest in the deterministic steady state —with $\beta \in (0,1)$ the household's subjective discount factor, π_t and Y_t are aggregate inflation and output in period t, and Y is aggregate output in the deterministic steady state. As a baseline, we consider Taylor (1993)'s original rule (henceforth, TR93) with parameters $\phi_\pi = 1.5$ and $\phi_y = 0.125$ (for quarterly data). In the analysis, we also experiment with different types of rules, including the SIT rule, Taylor–type rules augmented with a measure of financial fragility, and regime–contingent rules (see Section 6).

3.2 Households

The representative household is infinitely–lived and has preferences over a sequence of Dixit–Stiglitz consumption baskets of differentiated goods $\{C_t\}_{t=0}^{\infty}$ and a sequence of hours worked, $\{N_t\}_{t=0}^{\infty}$, represented by the expected intertemporal utility function

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \chi \frac{N_t^{1+\varphi}}{1+\varphi}\right)\right] \tag{2}$$

The parameters $\sigma \in \mathbb{R}_+$ and $\varphi \in \mathbb{R}_+$ denote, respectively, the inverse intertemporal elasticity of substitution and the inverse Frisch elasticity. Finally, $\mathbb{E}_t[\cdot]$ denotes the expectation operator conditional on the information set available at the end of period t.

At the beginning of period t, the household supplies labor N_t at a nominal wage rate W_t. At the end of period t, it purchases the consumption basket C_t at price P_t and invests in a private nominal bond in zero net supply B_{t+1} as well as in physical capital goods K_{t+1}. The household transfers these physical capital goods as equity to the firms at the end of the period.[13]

We assume that the household can freely transform units of the consumption basket C_t one–for–one into capital goods (and *vice versa*), implying that both items are valued at the same

[12]Given that there is no growth trend in our model, the term Y_t/Y corresponds to the GDP gap with respect to its long–run trend (or de–trended GDP) as defined in Taylor (1993)'s seminal paper. In Section E of the Online Appendix, we show that our analysis and results are robust to considering an alternative Taylor rule whereby the central bank reacts to expected —rather than current— inflation.

[13]The household can thus be seen as a venture capitalist that invests in a diversified portfolio of firm equity. In Section L.2.1 of the Online Appendix, we present a version of our model in which firms issue risk–free debt (*i.e.* debt that is not subject to moral hazard) and show that this version is isomorphic to our baseline one.

price, P_t. Among the K_{t+1} capital goods that the household purchases, $(1-\delta)K_t$ are old capital goods that have been used for production in period t, where $\delta \in (0,1)$ is the constant capital depreciation rate, and I_t are new capital goods produced in period t, with $K_{t+1} = (1-\delta)K_t + I_t$. At the time the household makes its investment/savings decisions, firms are all identical. As a consequence, the household transfers the same quantity of capital goods K_{t+1} to every firm or, equivalently, purchases the same amount of equity $P_t K_{t+1}$ from every firm.

The physical capital stock K_{t+1} (purchased in period t) yields a *state–contingent* real rate of return on equity r^q_{t+1} at the end of period $t+1$. In addition, the private nominal bond B_{t+1} yields a nominal interest rate i^b_t at the end of period $t+1$ *determined in period* t and defined as

$$i^b_t \equiv \frac{1+i_t}{Z_t} - 1 \tag{3}$$

where Z_t corresponds to a wedge between the private bond yield i^b_t and the policy rate i_t set by the central bank (as in Smets and Wouters (2007)). The wedge Z_t follows an exogenous AR(1) process $\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \varepsilon^z_t$ with $\rho_z \in (0,1)$ and $\varepsilon^z_t \rightsquigarrow N(0,\sigma^2_z)$ realized at the beginning of period t. Following the literature, we interpret Z_t as an aggregate demand shock.

In period t, the household's budget constraint is thus given by

$$P_t C_t + B_{t+1} + P_t K_{t+1} \leq W_t N_t + (1+i^b_{t-1})B_t + P_t(1+r^q_t)K_t + \Upsilon_t \tag{4}$$

where Υ_t is a lump–sum component of nominal income that includes retailers' dividends and lump–sum taxes.

The household determines its optimal consumption, labor supply, physical capital and bond holdings by maximizing its expected lifetime utility (2) subject to the sequence of budget constraints (4) for periods $t = 0, \ldots, \infty$. The first–order conditions describing the household's optimal behavior are standard and given by (alongside a transversality condition):

$$\frac{\chi N_t^{\varphi}}{C_t^{-\sigma}} = \frac{W_t}{P_t} \tag{5}$$

$$1 = (1+i^b_t)\mathbb{E}_t\left[\frac{\Lambda_{t,t+1}}{1+\pi_{t+1}}\right] \tag{6}$$

$$1 = \mathbb{E}_t\left[\Lambda_{t,t+1}\left(1+r^q_{t+1}\right)\right] \tag{7}$$

where the inflation rate π_{t+1} is defined as $\pi_{t+1} \equiv P_{t+1}/P_t - 1$ and $\Lambda_{t,t+k} \equiv \beta^k \left(C_{t+k}/C_t\right)^{-\sigma}$ denotes the stochastic discount factor between period t and $t+k$. Equation (5) states that the optimal labor supply behavior requires the marginal rate of substitution between consumption and leisure to be equal to the real wage. The no–arbitrage conditions (6) and (7) determine the optimal demands for bonds and physical capital (or firm equity).

The consumption basket of differentiated goods is defined by $C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i\right)^{\frac{\epsilon}{\epsilon-1}}$ where $C_t(i)$ is the final good i that the household purchases at price $P_t(i)$ for consumption purposes and ϵ denotes the demand elasticity (with $\epsilon > 1$). Likewise, new capital goods I_t are represented by the basket of differentiated goods $I_t \equiv \left(\int_0^1 I_t(i)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i\right)^{\frac{\epsilon}{\epsilon-1}}$ where $I_t(i)$ is the final good i that the household purchases at price $P_t(i)$ for investment purposes. Given total expenditures P_tC_t and P_tI_t, the household chooses the quantities of each differentiated good, $\{C_t(i)\}_{i\in[0,1]}$ and $\{I_t(i)\}_{i\in[0,1]}$, to maximize its consumption and investment baskets, C_t and I_t, subject to the constraints $\int_0^1 P_t(i)C_t(i)di = P_tC_t$ and $\int_0^1 P_t(i)I_t(i)di = P_tI_t$. This maximization yields the set of optimal demand schedules

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t \quad \text{and} \quad I_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} I_t$$

for all differentiated goods $i \in [0,1]$, as well as the aggregate price of the consumption/investment baskets $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon}\mathrm{d}i\right)^{\frac{1}{1-\epsilon}}$.

3.3 Retailers

Retailers are infinitely–lived and endowed with a linear production technology

$$Y_t(i) = X_t(i) \tag{8}$$

that transforms $X_t(i)$ units of the (single) intermediate good into $Y_t(i)$ units of a differentiated final good $i \in [0,1]$. Retailer i purchases intermediate goods on a perfectly competitive market and sells its differentiated good i in a monopolistically competitive environment. Accordingly, it sets its nominal price $P_t(i)$ by maximizing its intertemporal profit while taking into account the downward–sloping demand schedule for its differentiated good. Following Rotemberg (1982), we assume that when the retailer changes its price, it incurs a quadratic adjustment cost $\frac{\varrho}{2}P_tY_t\left(\frac{P_t(i)}{P_{t-1}(i)}-1\right)^2$, where $Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}}\mathrm{d}i\right)^{\frac{\epsilon}{\epsilon-1}}$ denotes aggregate output. In real terms, this adjustment cost takes the form of a basket of final goods similar to that of consumption and investment goods. The demand for the differentiated final good i thus emanates from the continuum of households (which consume and invest) and from other retailers (which incur the price adjustment cost). Accordingly, retailer i faces the demand schedule

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t \quad \forall i \in [0,1],\ \forall t \tag{9}$$

where $Y_t = C_t + I_t + \frac{\varrho}{2}Y_t\pi_t^2$, with $\frac{\varrho}{2}Y_t\pi_t^2$ being the aggregate real price adjustment cost in the symmetric equilibrium.

At the beginning of period t, retailer i chooses the price $P_t(i)$ that maximizes the market value of its current and future profits

$$\max_{P_t(i)} \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \Lambda_{t,t+k} \left[\frac{P_{t+k}(i)}{P_{t+k}} Y_{t+k}(i) - \frac{(1-\tau)p_{t+k}}{P_{t+k}} Y_{t+k}(i) - \frac{\varrho}{2} Y_{t+k} \left(\frac{P_{t+k}(i)}{P_{t+k-1}(i)} - 1 \right)^2 \right] \right\},$$

subject to the demand schedule (9), where p_{t+k} is the unit price of intermediate goods used as inputs in period $t+k$. Following standard practice, the purchase of intermediate goods is assumed to be subsidized at rate $\tau = 1/\epsilon$ to eliminate the steady–state distortions associated with monopolistic market power.

Since all retailers face the same price for the intermediate good and rely on the same transformation technology, the economy reaches a symmetric equilibrium in which $Y_{t+k}(i) = Y_{t+k}$ and $P_{t+k}(i) = P_{t+k}$. Retailers' optimal price–setting behavior yields the NK Phillips curve

$$(1+\pi_t)\pi_t = \mathbb{E}_t\left(\Lambda_{t,t+1}\frac{Y_{t+1}}{Y_t}(1+\pi_{t+1})\pi_{t+1}\right) - \frac{\epsilon-1}{\varrho}\left(\frac{\mathscr{M}_t - \mathscr{M}}{\mathscr{M}_t}\right) \tag{10}$$

where $\mathscr{M}_t$ is retailers' average markup given by

$$\mathscr{M}_t \equiv \frac{\epsilon}{\epsilon-1}\frac{P_t}{p_t} > 0 \tag{11}$$

and $\mathscr{M} \equiv \epsilon/(\epsilon-1)$ is the desired markup level that would prevail in the absence of nominal rigidities. Expression (10) implies that, all else equal, the inflation rate is positive when markups are below their desired level: in that case, retailers increase their prices to set their markups closer to their desired level.

3.4 Intermediate Goods Producers ("Firms")

The intermediate goods sector consists of overlapping generations of firms that are infinitesimally small, produce a homogeneous intermediate good and live for one period. The generation born at the end of one period dies at the end of the next period.[14]

Consider the generation of firms born at the end of period $t-1$. When they are born, these firms are identical and endowed with the same capital stock K_t as equity. As they enter period t, however, they face idiosyncratic productivity shocks. To cushion these shocks, firms may reallocate their capital goods among themselves through a secondary capital goods market while borrowing or lending to each other on an (intra–period) credit market.

Firms act in a perfectly competitive environment on all markets and, accordingly, take all prices as given. Technology and capital reallocation are described in turn.

3.4.1 Technology

Consider a firm born at the end of period $t-1$. When it is born, this firm obtains K_t units of capital goods from the household for use in period t. As it enters period t, the firm experiences the usual aggregate productivity shock as well as an idiosyncratic productivity shock that determines its access to a constant–returns–to–scale Cobb–Douglas technology. More specifically, we assume that a fraction μ of the firms are *unproductive* (with superscript u) and cannot produce anything, regardless of the quantity of capital and labor they use, while a fraction $1-\mu$

[14] As in Bernanke and Gertler (1989), Fuerst (1995), Bernanke et al. (1999), "generations" in our model should be thought of as representing the entry and exit of firms from the credit market, rather than as literal generations; a "period" in our model may therefore be interpreted as the length of a financial contract in this market. The overlapping–generation approach is standard in macroeconomic models because it provides a tractable framework for dynamic general equilibrium analysis with firm heterogeneity. In the presence of agency costs, this approach is a way to ignore multi–period financial contracts contingent on past debt repayments (see *e.g.* Gertler (1992) for an example of multi–period contracts in a three–period model). In Sections L.3 and L.4 of the Online Appendix, we discuss the robustness of our analysis when firms are infinitely lived or when they are heterogeneous *ex ante* (*i.e.* even before they incur the idiosyncratic productivity shocks).

are *productive* (with superscript p) and can produce

$$X_t^p = A_t (K_t^p)^\alpha (N_t^p)^{1-\alpha} \tag{12}$$

units of the intermediate good with K_t^p units of capital and N_t^p units of labor. Aggregate productivity A_t evolves randomly according to a stationary AR(1) process $\ln(A_t) = \rho_a \ln(A_{t-1}) + \varepsilon_t^a$ with $\rho_a \in (0,1)$ and $\varepsilon_t^a \rightsquigarrow N(0, \sigma_a^2)$, where the innovation ε_t^a is realized at the beginning of period t.

We assume that both unproductive and productive firms have access to a storage technology for their capital, allowing them to keep their capital idle throughout period t. In that case, capital depreciates at the same rate (or must be maintained at the same cost) δ as when it is used productively. Hence, the "rate of return" on storage for both types of firms is $-\delta$.[15]

3.4.2 Capital Reallocation Through a Credit Market

We assume that all firms have free and equal access to a secondary capital goods market as well as to a credit market, through which they can "borrow or lend capital goods" to each other. Upon observing its type, a productive firm can buy $K_t^p - K_t$ additional capital goods on credit from unproductive firms to increase the quantity of capital goods used in production from K_t to K_t^p. Symmetrically, an unproductive firm may sell $K_t - K_t^u$ capital goods on credit to productive firms, to reduce the quantity of capital goods it keeps idle from K_t to K_t^u.

The secondary capital goods market thus operates in lockstep with the credit market, which helps to reallocate capital to productive firms. In Section 3.5.1 we will show that, absent financial frictions, the credit market fully corrects the initial capital misallocation. In that case, our model boils down to the baseline NK model with endogenous capital accumulation and one representative intermediate goods producer.

The credit market is perfectly competitive and firms (which are infinitesimally small) take the real interest rate on this market as given. Let r_t^c denote this interest rate. A firm that borrows capital at the beginning of period t must repay $1 + r_t^c$ final goods per unit of borrowed capital to its lenders at the end of period t. Meanwhile, a firm that lends capital at the beginning of period t will receive $1 + r_t^c$ final goods at the end of period t per unit of loaned capital.

Frictions in the Credit Market. We introduce two frictions in this credit market. The first friction is asymmetric information. We assume that lenders do not observe whether a prospective borrower is a productive or an unproductive firm. As a consequence, lenders lend the same amount to all types of firms, and unproductive firms may mimic productive firms by borrowing $K_t^p - K_t$. To economize on notation, we henceforth denote by $K_t^p - K_t$ the loan that *every* borrower —whether productive or unproductive— demands in the credit market.

The second credit friction is moral hazard. We assume that idle capital cannot be seized freely by lenders. Instead of employing the borrowed capital goods in production, a borrower

[15] This assumption implies that the return on capital is always higher for a productive firm that produces than for a firm that stores its capital (as shown in relation (20) derived below).

may therefore keep them idle, sell them at the end of the period, abscond with the proceeds, and default. More formally, a firm that borrows $K_t^p - K_t > 0$ capital goods may abscond with its own capital K_t plus a fraction $1-\theta$ of the borrowed capital, with $\theta < 1$. Since idle capital is stored and depreciates at rate δ, a borrower that absconds and defaults earns $(1-\delta)K_t + (1-\theta)(1-\delta)(K_t^p - K_t)$ —which is more than the payoff from storing its own capital stock only. The remainder of the undepreciated borrowed capital, $\theta(1-\delta)(K_t^p - K_t)$, is assumed to be partly recouped by lenders and partly dissipated in debt–collection proceedings.

By contrast, we assume that a (productive) firm that produces intermediate goods cannot abscond and always repays its debt at the end of the period. In other words, once it has produced intermediate goods, a (productive) firm cannot default. One can think of such a firm as one that operates transparently (*e.g.* sets up a business, hires labor, etc.) and whose cash–flow from the sale of intermediate goods cannot be concealed from lenders.[16]

Together, these two credit frictions allow firms to divert funding from its intended use: to boost their profits, firms may want to borrow, purchase more capital, abscond, and default. In what follows, we focus on the case where lenders never allow a firm to borrow if there is a possibility that the firm defaults down the line.[17]

Borrowing Limit. To deter borrowers from defaulting, lenders impose a borrowing limit on each borrower. Since lenders do not observe borrowers' types, this limit is the same for all borrowers. Lenders set the borrowing limit such that no firm —whether productive or unproductive— wants to borrow, divert the borrowed capital, and default. Recalling that both unproductive and productive firms have the option to lend capital goods at the rate r_t^c, no firm will ever borrow and default if the payoff of doing so is smaller than the payoff from lending capital goods.[18] The incentive–compatibility constraint that ensures that no firm defaults thus reads:

$$\underbrace{(1-\delta)K_t + (1-\theta)(1-\delta)(K_t^p - K_t)}_{\text{payoff if the firm borrows and defaults}} \leq \underbrace{(1+r_t^c)K_t}_{\text{payoff if the firm lends}} \tag{13}$$

Proposition 1 follows.

Proposition 1. ***(Incentive–Compatible Borrowing Limit)*** *Given the credit market rate r_t^c,*

[16]If the firms that produce intermediate goods also had the possibility to abscond and default, then no lender would ever want to lend. To make things interesting, we assume that the proceeds of the sale of intermediate goods can be freely seized by lenders and therefore that the firms that produce intermediate goods never default.

[17]In Section J of the Online Appendix, we study a general version of the model where lenders may allow borrowers to default, thus potentially giving rise to pooling equilibria in which some unproductive firms also borrow and default in some states of Nature. In those states of Nature, lenders would typically charge a risk premium that compensates them for loan losses. When lenders incur debt–collection costs, this risk premium increases with the probability of default, the loss given default, and the debt–collection costs. We then show that, under a plausible parametrization of these costs (*i.e.* provided that they are not unrealistically small), in the states of Nature where lenders would be willing to tolerate defaults, the risk premium that they would have to charge to make up for their losses is always too high for the productive borrowers to afford a loan. As a result, lenders do not lend and the credit market shuts down in equilibrium. Therefore, under a realistic parametrization of the model, borrower defaults *per se* are *off–equilibrium* outcomes. For simplicity, we abstract from equilibria with defaults (*i.e.* pooling equilibria) from the outset.

[18]Productive firms have the additional option to borrow and produce, which unproductive firms do not have.

a firm cannot borrow and purchase more than a fraction of its initial capital stock:

$$\frac{K_t^p - K_t}{K_t} \leq \max\left\{\frac{r_t^c + \delta}{(1-\delta)(1-\theta)}; 0\right\} \tag{IC}$$

Proof. The result follows from (13) and the non–negativity of the borrowing limit, hence the "max" operator on the right–hand side of (IC). □

As long as condition (IC) is satisfied, all firms will refrain from borrowing and defaulting. The ratio on the right–hand side summarizes the firm's trade–off between lending, with return $r_t^c + \delta$, and borrowing with default, with return $(1-\theta)(1-\delta)$. The borrowing limit increases with the loan rate r_t^c: the higher the loan rate, the higher firms' opportunity cost of defaulting, the less likely it is that firms borrow and abscond in search for yield, and the higher the incentive–compatible borrowing limit. Finally, note that when $\theta = 1$ (*i.e.* when borrowers cannot abscond with the borrowed capital), the moral hazard problem vanishes and lenders do not impose any borrowing limit at all (the term on the right side of (IC) is infinite).

We now turn to the analysis of productive and unproductive firms' respective profit maximization problems.

3.4.3 Optimization Problem of a Productive Firm

The productive firms' problem consists of choosing production X_t^p and the associated quantities of capital K_t^p, labor N_t^p, and credit $K_t^p - K_t$ to maximize their profit and dividend payouts to the household, subject to the borrowing limit (IC).

At the end of period t, a productive firm sells its production X_t^p to retailers at price p_t, pays its workers $W_t N_t^p$, sells its undepreciated capital $(1-\delta)K_t^p$ at price P_t, repays its loan (in nominal terms) $P_t(1+r_t^c)(K_t^p - K_t)$ to lenders (given (IC)), and distributes its profits as dividends to the representative household. Let D_t^p and $r_t^{q,p}$ denote the dividends (in nominal terms) and the real rate of return on equity of a productive firm, respectively. The firm's optimization problem can therefore be written as:

$$\begin{aligned}\max_{K_t^p, N_t^p} D_t^p \equiv P_t(1+r_t^{q,p})K_t &= p_t X_t^p - W_t N_t^p + P_t(1-\delta)K_t^p - P_t(1+r_t^c)(K_t^p - K_t) \\ &= \underbrace{p_t X_t^p - W_t N_t^p - P_t(r_t^c+\delta)K_t^p}_{\substack{\text{excess return from borrowing} \\ K_t^p - K_t \text{ relative to lending } K_t}} + P_t(1+r_t^c)K_t\end{aligned} \tag{14}$$

with respect to K_t^p and N_t^p, subject to the borrowing constraint (IC). The second line in equation (14) highlights that a productive firm earns an excess return if it borrows (rather than lends). Given relation (11), the firm's objective in (14) boils down to maximizing the return on equity

$$\max_{K_t^p, N_t^p} r_t^{q,p} = \frac{\epsilon}{\epsilon - 1}\frac{X_t^p}{\mathscr{M}_t K_t} - \frac{W_t}{P_t}\frac{N_t^p}{K_t} - (r_t^c + \delta)\frac{K_t^p - K_t}{K_t} - \delta \tag{15}$$

The optimal labor demand N_t^p satisfies the first–order condition

$$\frac{W_t}{P_t} = \frac{\epsilon}{\epsilon - 1}\frac{(1-\alpha)X_t^p}{\mathscr{M}_t N_t^p} = \frac{\epsilon}{\epsilon - 1}(1-\alpha)\frac{A_t}{\mathscr{M}_t}\left(\frac{K_t^p}{N_t^p}\right)^\alpha \tag{16}$$

Let

$$\Phi_t \equiv \alpha \frac{X_t^p}{K_t^p} = \alpha A_t \left(\frac{N_t^p}{K_t^p}\right)^{1-\alpha} \tag{17}$$

denote the marginal product of capital for a productive firm. Using (16), one can further express Φ_t as a function of the real wage W_t/P_t and retailers' markup $\mathscr{M}_t$,

$$\Phi_t = \alpha A_t^{\frac{1}{\alpha}} \left(\frac{\epsilon}{\epsilon - 1} \frac{1-\alpha}{\mathscr{M}_t \frac{W_t}{P_t}}\right)^{\frac{1-\alpha}{\alpha}} \tag{18}$$

which the firm takes as given. Using (16) and (17), the firm's optimization problem in (15) can be further simplified to

$$\max_{K_t^p} r_t^{q,p} = r_t^c + \left(r_t^k - r_t^c\right) \frac{K_t^p}{K_t} \tag{19}$$

subject to the borrowing limit (IC), where r_t^k denotes the rate of return on capital (after capital depreciation), defined by:

$$r_t^k \equiv \frac{\epsilon}{\epsilon - 1} \frac{\Phi_t}{\mathscr{M}_t} - \delta > -\delta \tag{20}$$

Expression (19) shows that a productive firm takes its decision based on the gap between the rate of return on capital r_t^k and the loan market rate r_t^c —both of which it takes as given. The optimal quantity of physical capital K_t^p that the firm uses to produce and the corresponding amount of credit $K_t^p - K_t$ will be determined later as we derive the interest rate r_t^c that clears the credit market.

3.4.4 Optimization Problem of an Unproductive Firm

Given the borrowing limit (IC), unproductive firms choose between selling their capital and lending the proceeds at the real loan rate r_t^c, and keeping their capital idle and selling it at price P_t at the end of the period. Let K_t^u be the quantity of capital goods that an unproductive firm keeps idle and stores, D_t^u its dividends (in nominal terms), and $r_t^{q,u}$ its real rate of return on equity. The optimization problem of this firm consists of maximizing its dividend payouts:

$$\begin{aligned} \max_{K_t^u} D_t^u \equiv P_t(1 + r_t^{q,u})K_t &= P_t(1-\delta)K_t^u + P_t(1 + r_t^c)(K_t - K_t^u) \\ &= \underbrace{P_t(-\delta - r_t^c)K_t^u}_{\substack{\text{forgone return from} \\ \text{keeping } K_t^u \text{ capital goods idle}}} + P_t(1 + r_t^c)K_t \end{aligned} \tag{21}$$

with respect to K_t^u, where $K_t - K_t^u$ is the quantity of goods that the firm sells/lends. The above maximization problem can be further simplified to:

$$\max_{K_t^u} r_t^{q,u} = r_t^c + (-\delta - r_t^c) \frac{K_t^u}{K_t} \tag{22}$$

which shows that an unproductive firm takes its decision based on the gap between the net return on storage $-\delta$ and the loan market rate r_t^c. The optimal quantity of physical capital K_t^u that the firm keeps idle will be determined in the next section, as we derive the interest rate r_t^c that clears the credit market.

3.5 Credit–Market Equilibrium

We focus on perfectly competitive, or Walrasian, equilibria in which firms are infinitesimal and take the credit–market rate r_t^c as given:

Definition 1. ***(Competitive Equilibrium)*** *A credit–market equilibrium is characterized by individual loan demands $K_t^p - K_t$ and supplies $K_t - K_t^u$ and a loan rate r_t^c such that:*

(i) given r_t^c, each firm determines its individual loan demand/supply so as to maximize its return on equity (see (19) and (22)) subject to the borrowing limit (IC);

(ii) given the sum of individual optimal loan demands/supplies, r_t^c clears the credit market.

We derive the credit–market equilibrium in two steps. In the first step, we consider the *partial* equilibrium case in which we take the rest of the economy and, in particular, productive firms' rate of return on capital r_t^k as given. We derive the equilibrium outcome in the absence (Section 3.5.1) as well as in the presence (Section 3.5.2) of financial frictions. In the second step, we study the credit market outcome in the *general* equilibrium in the presence of financial frictions (Section 3.5.3). Solving the general equilibrium necessitates acknowledging that r_t^k is endogenous and depends on the credit–market equilibrium outcome.

3.5.1 Credit Market Partial Equilibrium: Frictionless Case

This section studies the partial equilibrium of the credit market in a frictionless case where lenders can observe borrowers' types and enforce loan repayments. In that case firms do not face the borrowing limit (IC) and can borrow freely any desired amount.

Let $L^S(r_t^c)$ denote the aggregate supply of credit by unproductive firms. Given their optimization problem in (22), these firms sell their entire capital stock K_t and lend the proceeds of the sale when $r_t^c > -\delta$, implying $K_t^u = 0$. Since there is a mass μ of unproductive firms, one obtains $L^S(r_t^c) = \mu K_t$. When $r_t^c = -\delta$, these firms are indifferent between selling (units of) their initial capital stock and lending the proceeds in the credit market, doing nothing, or borrowing in the credit market to buy additional units of capital and keep them idle: $K_t^u \in [0, +\infty)$ and $L^S(r_t^c) \in (-\infty, \mu K_t]$. When $r_t^c < -\delta$, unproductive firms borrow as much as possible to buy capital goods and keep them idle: $K_t^u = +\infty$ and $L^S(r_t^c) = -\infty$. The aggregate credit supply (by unproductive firms) is therefore given by

$$L^S(r_t^c) = \mu\left(K_t - K_t^u\right) = \begin{cases} \mu K_t & \text{for } r_t^c > -\delta \\ (-\infty, \mu K_t] & \text{for } r_t^c = -\delta \\ -\infty & \text{for } r_t^c < -\delta \end{cases}$$

and is represented by the black line in Figure 2.

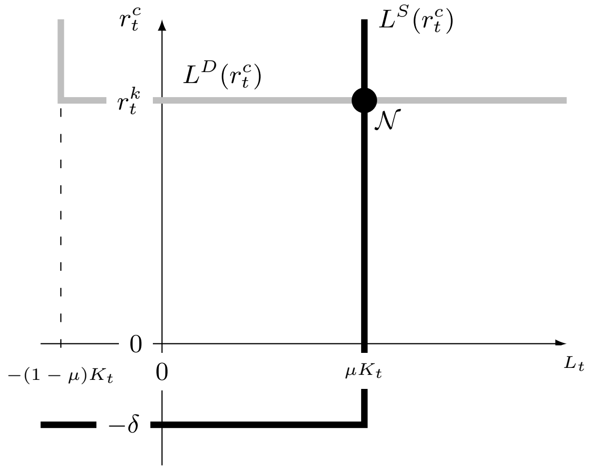
Let $L^D(r_t^c)$ denote the aggregate demand for credit by productive firms. In the absence of any borrowing constraint, and given their optimization problem in (19), these firms borrow as much as possible to buy additional units of capital when $r_t^c < r_t^k$, implying $K_t^p = +\infty$ and $L^D(r_t^c) = +\infty$.

When $r_t^c = r_t^k$, productive firms are indifferent between borrowing and lending. Since there is a mass $1-\mu$ of productive firms, one obtains $K_t^p \in [0, +\infty)$ and $L^D(r_t^c) \in [-(1-\mu)K_t, +\infty)$. When $r_t^c > r_t^k$, these firms sell their entire capital stock K_t and lend the proceeds of the sale: $K_t^p = 0$ and $L^D(r_t^c) = -(1-\mu)K_t$. The aggregate credit demand (from productive firms) is therefore given by

$$L^D(r_t^c) = (1-\mu)\left(K_t^p - K_t\right) = \begin{cases} -(1-\mu)K_t & \text{for } r_t^c > r_t^k \\ [-(1-\mu)K_t, +\infty) & \text{for } r_t^c = r_t^k \\ +\infty & \text{for } r_t^c < r_t^k \end{cases}$$

and is represented by the gray line in Figure 2.

Figure 2: Frictionless Credit Market



Notes: This figure illustrates unproductive firms' aggregate credit supply (black curve) and productive firms' aggregate credit demand (gray curve) in the absence of financial frictions. This figure abstracts from general equilibrium feedback effects, *i.e.* takes r_t^k as given with $r_t^k > -\delta$, as implied by relation (20).

Figure 2 shows that there is one unique equilibrium $\mathcal{N}$, with $r_t^c = r_t^k > -\delta$ and $K_t^u = 0$, implying that $r_t^{q,p} = r_t^{q,u} = r_t^k = r_t^c$. Hence, all firms have the same rate of return on equity. As the unproductive firms (of mass μ) lend their entire capital stock K_t to productive firms (of mass $1-\mu$), the equilibrium is also characterized by a perfect reallocation of the capital stock from unproductive to productive firms:

$$(1-\mu)K_t^p = K_t \tag{23}$$

In this case, our model boils down to the textbook NK model with endogenous capital accumulation and one representative intermediate goods producer.

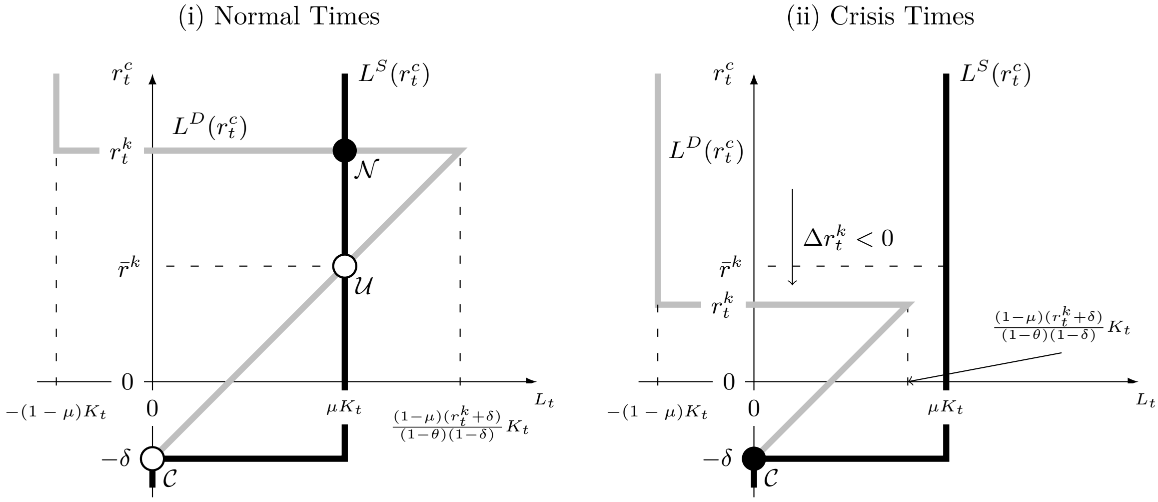
3.5.2 Credit Market Partial Equilibrium: Frictional Case

Consider now the frictional case where lenders face asymmetric information and moral hazard. In that case, lenders impose the borrowing limit (IC) on borrowers so that unproductive firms never borrow. The equilibrium of the credit market is illustrated in Figure 3.

As in the frictionless credit case, the supply of credit emanates from the unproductive firms. When $r_t^c > -\delta$, unproductive firms (of mass μ) sell their capital stock K_t and lend the proceeds in the credit market, implying $L^S(r_t^c) = \mu K_t$. When $r_t^c = -\delta$, unproductive firms are indifferent between lending and keeping their capital idle, $L^S(r_t^c) \in [0, \mu K_t]$. When $r_t^c < -\delta$, they keep their capital idle: $L^S(r_t^c) = 0$. As a result, the aggregate credit supply curve $L^S(r_t^c)$, represented by the black line in Figure 3, is given by:

$$L^S(r_t^c) = \mu\left(K_t - K_t^u\right) = \begin{cases} \mu K_t & \text{for } r_t^c > -\delta \\ [0, \mu K_t] & \text{for } r_t^c = -\delta \\ 0 & \text{for } r_t^c < -\delta \end{cases} \tag{24}$$

Figure 3: Frictional Credit Market



Notes: This figure illustrates unproductive firms' aggregate credit supply (black curve) and productive firms' incentive–compatible aggregate credit demand (gray curve). In panel (i), the demand curve is associated with a value of r_t^k strictly above $\bar{r}^k$ and multiple equilibria $\mathscr{C}$, $\mathscr{N}$, and $\mathscr{U}$. In this case, $\mathscr{U}$ and $\mathscr{C}$ are ruled out on the grounds that they are unstable (for $\mathscr{U}$) and Pareto–dominated (for $\mathscr{C}$). In panel (ii), the demand curve is associated with a value of r_t^k strictly below $\bar{r}^k$ and $\mathscr{C}$ is the unique equilibrium. The threshold for the loan rate, $\bar{r}^k$, is constant and corresponds to the minimum incentive–compatible loan rate that is required to ensure that none of the firms borrows and absconds. This figure abstracts from general equilibrium feedback effects, *i.e.* takes r_t^k as given with $r_t^k > -\delta$, as implied by relation (20).

Likewise, the demand for credit emanates from the productive firms. When $r_t^c > r_t^k$, productive firms prefer to sell their capital and lend the proceeds rather than borrow: $L^D(r_t^c) = -(1-\mu)K_t$. When $r_t^c = r_t^k$, productive firms are indifferent between lending and borrowing up to the borrowing limit in (IC), implying $L^D(r_t^c) \in [-(1-\mu)K_t, \max\{(1-\mu)(r_t^k+\delta)K_t/(1-\delta)(1-\theta); 0\}]$. When $r_t^c < r_t^k$, they borrow up to the limit, implying $L^D(r_t^c) = \max\{(1-\mu)(r_t^k+\delta)K_t/(1-\delta)(1-\theta); 0\}$. As a result, the aggregate credit demand curve $L^D(r_t^c)$ (Figure 3, gray line) is given by:

$$L^D(r_t^c) = (1-\mu)\left(K_t^p - K_t\right) = \begin{cases} -(1-\mu)K_t & \text{for } r_t^c > r_t^k \\ \left[-(1-\mu)K_t, \max\left\{\frac{(1-\mu)(r_t^c+\delta)}{(1-\delta)(1-\theta)}K_t; 0\right\}\right] & \text{for } r_t^c = r_t^k \\ \max\left\{\frac{(1-\mu)(r_t^c+\delta)}{(1-\delta)(1-\theta)}K_t; 0\right\} & \text{for } r_t^c < r_t^k \end{cases} \tag{25}$$

The credit–market equilibrium corresponds to a situation where the credit supply and demand curves intersect (see Definition 1). Figure 3 shows that three types of equilibrium may exist in the credit market: $\mathscr{N}$, $\mathscr{U}$, and $\mathscr{C}$. We discuss these equilibria in turn.

Equilibria $\mathscr{N}$ and $\mathscr{U}$ exist only in some states of Nature and, whenever they exist, coexist. In equilibrium $\mathscr{N}$, $r_t^c = r_t^k$ and the μ unproductive firms sell their capital to the $1-\mu$ productive ones, as if there were no financial frictions: equilibrium $\mathscr{N}$ in panel (i) of Figure 3 is the same as equilibrium $\mathscr{N}$ in Figure 2. In that case, there is perfect capital reallocation, with $K_t^u = 0$ and $K_t^p = K_t/(1-\mu)$. In equilibrium $\mathscr{U}$, $r_t^c < r_t^k$ but capital is also perfectly reallocated from unproductive to productive firms and the *aggregate* outcome is the same as in equilibrium $\mathscr{N}$. Equilibrium $\mathscr{U}$ differs from $\mathscr{N}$ on two counts. First, the rates of return on equity of unproductive and productive firms are not equal in $\mathscr{U}$ (where $r_t^{q,u} < r_t^{q,p}$), whereas they are equal in $\mathscr{N}$. Second, $\mathscr{U}$ is not stable under tâtonnement (see Mas-Colell et al. (1995), Chapter 17). For this reason (and without loss of generality), we henceforth focus on equilibrium $\mathscr{N}$ and refer to this equilibrium as "normal times".

Equilibrium $\mathscr{C}$ exists in all states of Nature. In equilibrium $\mathscr{C}$, $r_t^c = -\delta$ and unproductive firms are indifferent between keeping their capital idle or selling it and lending the proceeds.[19] Hence, any supply of funds within the interval $[0, \mu K_t]$ is consistent with optimal firm behavior. Meanwhile, the incentive–compatible quantity of capital goods that a firm can borrow at that rate is zero (see (IC)), and there is no demand. As a result, $K_t^u = K_t^p = K_t$ and $L^D(-\delta) = L^S(-\delta) = 0$. Equilibrium $\mathscr{C}$ involves no trade and no capital reallocation. We refer to this autarkic equilibrium as a "financial crisis".

The equilibrium outcome determines the extent of capital reallocation in the economy and, more particularly, the fraction of the economy's capital stock that productive firms use for the production of intermediate goods. Let ω_t denote this fraction:

$$\omega_t \equiv \frac{(1-\mu)K_t^p}{K_t} \tag{26}$$

The value of ω_t can be seen as a measure of capital reallocation in the economy. In normal times, there is perfect capital reallocation, the entire capital stock is used productively, and $\omega_t = 1$. In a crisis, in contrast, there is no capital reallocation, the mass μ of unproductive firms keep their capital idle and only a fraction $1-\mu$ of the economy's aggregate capital stock is used for production: $\omega_t = 1-\mu$. Hence:

$$\omega_t = \begin{cases} 1 & \text{in normal times (Equilibrium } \mathscr{N}) \\ 1-\mu & \text{in crisis times (Equilibrium } \mathscr{C}) \end{cases} \tag{27}$$

3.5.3 Credit–Market Outcome in the General Equilibrium

In the general equilibrium, the fraction ω_t of the economy's capital stock that is used by the productive firms affects these firms' rate of return on capital r_t^k. Variations of ω_t affect the

[19] Absent borrower defaults, the equilibrium loan rate r_t^c is *risk–free*. Note, however, that a crisis in our model corresponds to a situation where default risk is so high that the risk premium that unproductive firms would need to charge if they lent is above the premium that productive firms can afford to pay. Since productive firms do not want to borrow in that case, unproductive firms do not lend. In our model, the collapse of the credit market during a crisis should thus be seen as the flip side of prohibitively high *off–equilibrium* default risk premia. For a detailed discussion on the possibility that lenders tolerate defaults and the attendant risk premium, see Section J of the Online Appendix.

demand for labor and the supply of intermediate goods and, therefore, the real wage W_t/P_t, the intermediate goods prices p_t, the markup $\mathscr{M}_t$, and eventually r_t^k —as the expressions of Φ_t and r_t^k in (18) and (20) suggest. This is the case in any given state of Nature $\{A_t, Z_t, K_t\}$. To emphasize that r_t^k varies with ω_t, it will be useful to express r_t^k as a function of ω_t:

$$r_t^k = r^k\left(\omega_t \mid A_t, Z_t, K_t\right) \tag{28}$$

whenever such emphasis is needed. Note that the exact form of this function can only be pinned down numerically as one solves for the dynamic general equilibrium. Proposition 2 follows.

Proposition 2. ***(Existence of a credit–market equilibrium with Trade $\mathscr{N}$)*** *Consider the state of Nature $\{A_t, Z_t, K_t\}$ and let $r^k\left(\omega_t = 1 \mid A_t, Z_t, K_t\right)$ be the productive firms' rate of return on capital that prevails in the general equilibrium if $\omega_t = 1$. Then, equilibrium $\mathscr{N}$ exists if and only if*

$$r^k\left(\omega_t = 1 \mid A_t, Z_t, K_t\right) \geq \bar{r}^k \equiv \frac{(1-\theta)(1-\delta)\mu}{1-\mu} - \delta \tag{29}$$

Whenever equilibrium $\mathscr{N}$ exists, it always coexists with the crisis equilibrium $\mathscr{C}$.

Proof. The proof consists of two stages. The first stage considers the partial equilibrium: given r_t^k, a competitive credit–market equilibrium with trade $\mathscr{N}$ exists only if there is a loan rate r_t^c at which the loan demand schedule (Figure 3, panel (i), gray line) intersects the supply schedule (black line) for a strictly positive amount of credit, *i.e.* only if $\lim_{r_t^c \nearrow r_t^k} L^D(r_t^c) \geq L^S(r_t^k)$. Using relations (IC), (24) and (25), this condition can be rewritten as $(1-\mu)(r_t^k+\delta)/(1-\theta)(1-\delta) \geq \mu \Leftrightarrow r_t^k \geq (1-\theta)(1-\delta)\mu/(1-\mu) - \delta$. When this condition is satisfied, the credit market reallocates unproductive firms' entire capital stock to the productive firms and $\omega_t = 1$. The second stage considers the general equilibrium for a given state of Nature $\{A_t, Z_t, K_t\}$: since r_t^k varies with ω_t, equilibrium $\mathscr{N}$ exists if and only if the above condition is satisfied for the value of r_t^k that prevails in the general equilibrium if $\omega_t = 1$, *i.e.* for $r_t^k = r^k\left(\omega_t = 1 \mid A_t, Z_t, K_t\right)$. Finally, since the autarkic equilibrium $\mathscr{C}$ always exists, it necessarily coexists with $\mathscr{N}$ whenever the latter equilibrium exists. □

The interest rate threshold $\bar{r}^k$ in Proposition 2 is the minimum return on capital that guarantees the existence of an equilibrium with trade and (full) capital reallocation. Perhaps more intuitively, $\bar{r}^k$ can also be seen as the minimum loan rate that unproductive firms require to lend in the credit market rather than borrow funds and abscond in search for yield. To see this, notice that borrowers' incentive compatibility constraint (13) can be rewritten as a condition on the loan rate: $r_t^c \geq (1-\theta)(1-\delta)(K_t^p - K_t)/K_t - \delta$, which simply means that an unproductive firm has an incentive to lend only if the loan rate is high enough. For this condition to be satisfied in an equilibrium with trade, *i.e.* when $\mu K_t = (1-\mu)(K_t^p - K_t)$, one must therefore have $r_t^c \geq \bar{r}^k \equiv (1-\theta)(1-\delta)\mu/(1-\mu) - \delta$, which corresponds to the right–hand side of relation (29). Further, notice that productive firms only borrow funds if their return $r_t^k = r^k\left(\omega_t = 1 \mid A_t, Z_t, K_t\right)$ in equilibrium $\mathscr{N}$ is higher than their cost of funds r_t^c (see (19)). When $r_t^k < \bar{r}^k \leq r_t^c$, productive firms cannot afford to pay the minimum loan rate that unproductive firms require, and the credit–market equilibrium with trade in $\mathscr{N}$ cannot emerge. In that case, equilibrium $\mathscr{C}$ is the only one that survives and the credit market collapses.

In what follows, we rule out the possibility of "self–fulfilling" (*i.e.* non–fundamental) equilibria by assuming that firms always coordinate on the equilibrium associated with the highest volume of trade and capital reallocation whenever multiple equilibria coexist:[20]

Assumption 1. ***(Uniqueness of the Competitive Equilibrium)*** *There is no coordination failure: whenever equilibria $\mathscr{C}$ and $\mathscr{N}$ coexist, firms always coordinate on equilibrium $\mathscr{N}$, which is associated with the highest fraction ω_t of the capital stock used productively —i.e. with the most efficient capital reallocation.*

Proposition 2 and Assumption 1 require that the general equilibrium be solved sequentially. In a first stage, we assume that equilibrium $\mathscr{N}$ exists and solve the general equilibrium for the return on capital $r_t^k = r^k\left(\omega_t = 1 \mid A_t, Z_t, K_t\right)$ that is consistent with a perfect reallocation of capital (*i.e.* $\omega_t = 1$). In a second stage, we verify that $r^k\left(\omega_t = 1 \mid A_t, Z_t, K_t\right) \geq \bar{r}^k$. If this condition is satisfied, we conclude that equilibrium $\mathscr{N}$ exists and select it. Otherwise, we conclude that $\mathscr{C}$ is the only possible equilibrium in the general equilibrium. In Section K of the Online Appendix, we check that equilibria $\mathscr{N}$ and $\mathscr{C}$ are robust to price deviations by an individual firm.

3.6 Aggregate Outcome

3.6.1 Market Clearing Conditions

Since only the productive firms produce, the labor, intermediate goods, and final goods markets clear when

$$N_t = (1-\mu)N_t^p \tag{30}$$

$$Y_t = (1-\mu)X_t^p \tag{31}$$

$$Y_t = C_t + I_t + \frac{\varrho}{2}Y_t\pi_t^2 \tag{32}$$

3.6.2 Aggregate Output

Aggregate output depends on the quantity $\omega_t K_t$ of capital goods that are used productively:

Corollary 1. ***(Aggregate Output and Productivity)***

$$Y_t = A_t\left(\omega_t K_t\right)^\alpha N_t^{1-\alpha} = \hat{A}_t K_t^\alpha N_t^{1-\alpha}, \text{ with } \hat{A}_t \equiv A_t\omega_t^\alpha \text{ and } \omega_t \in \{1-\mu, 1\} \tag{33}$$

Proof. Combine relations (12), (26), (30) and (31). □

[20]There are, of course, several —but less parsimonious— ways to select the equilibrium. For example, one could introduce a sunspot, *e.g.* assume that firms coordinate on equilibrium $\mathscr{N}$ (*i.e.* are "optimistic") with some constant and exogenous probability whenever this equilibrium exists. It should be clear, however, that the central element of our analysis is Proposition 2 for the existence of $\mathscr{N}$, and not the selection of $\mathscr{N}$ conditional on its existence. In other words, our analysis does not hinge on the assumed equilibrium selection mechanism.

Relation (33) emphasizes the direct link between credit–market functioning, as captured by ω_t, and aggregate productivity $\hat{A}_t$: the higher the amount of capital reallocated through the credit market, the higher the fraction ω_t of the capital stock that is used by productive firms, and the higher aggregate productivity. Since $\omega_t = 1$ in normal times and $\omega_t = 1-\mu$ in a crisis, the latter induces (all else equal) a discrete fall in aggregate productivity and output by a fraction $1-(1-\mu)^\alpha$.

3.6.3 Household's Return on Equity

Since the household fully diversifies its portfolio of equity investments across firms at the end of period $t-1$, its overall return on equity r_t^q is equal to the weighted sum of unproductive and productive firms' respective returns on equity at the end of period t:

$$r_t^q = \mu r_t^{q,u} + (1-\mu) r_t^{q,p}$$

Substituting expressions (19) and (22) in the expression of r_t^q above, one obtains:

$$r_t^q = \mu\left(r_t^c + (-\delta - r_t^c)\frac{K_t^u}{K_t}\right) + (1-\mu)\left(r_t^c + (r_t^k - r_t^c)\frac{K_t^p}{K_t}\right)$$

In normal times, unproductive firms do not store capital ($K_t^u = 0$), productive firms operate with $K_t^p = K_t/(1-\mu)$, and the credit market loan rate is equal to productive firms' return on capital ($r_t^c = r_t^k$), implying $r_t^q = r_t^k$. In crisis times, there is no capital reallocation, $K_t^p = K_t^u = K_t$, and $r_t^c = -\delta$, which implies that $r_t^q = \mu\cdot(-\delta) + (1-\mu)r_t^k$. One can therefore write:

$$r_t^q = (1-\omega_t)\cdot(-\delta) + \omega_t r_t^k = \omega_t(r_t^k + \delta) - \delta \tag{34}$$

where $\omega_t \in \{1-\mu, 1\}$. The first equality highlights that r_t^q can also be interpreted as the *aggregate* return on capital, which is equal to the weighted average of the returns on idle capital goods ($-\delta$) and productive capital goods (r_t^k), using as weights the fractions $1-\omega_t$ and ω_t of idle and productive capital, respectively. In addition, relations (17) and (20) imply $r_t^k + \delta = \frac{\epsilon}{\epsilon-1}\frac{\alpha X_t^p}{\mathscr{M}_t K_t^p}$ which, using (26) and (31), further yields

$$r_t^k = \frac{\epsilon}{\epsilon-1}\frac{\alpha Y_t}{\mathscr{M}_t(\omega_t K_t)} - \delta \tag{35}$$

Substituting the expression for r_t^k above into (34), one finally obtains the household's return on equity as a function of the economy's aggregates:

$$r_t^q = \frac{\epsilon}{\epsilon-1}\frac{\alpha Y_t}{\mathscr{M}_t K_t} - \delta \tag{36}$$

where Y_t and $\mathscr{M}_t$ are consistent with the value of $\omega_t \in \{1-\mu, 1\}$ in the general equilibrium.

4 Transmission Channels

What are the channels through which monetary policy affects financial stability? To answer this question, it is useful to unpack condition (29) using relation (35) for $\omega_t = 1$. One obtains:

Corollary 2. ***(Monetary Policy and Financial Stability)*** *The normal–times equilibrium $\mathscr{N}$ exists if and only if*

$$\frac{\epsilon}{\epsilon-1}\frac{\alpha Y_t}{\mathscr{M}_t K_t} \geq \frac{(1-\theta)(1-\delta)\mu}{1-\mu}$$

where Y_t and $\mathscr{M}_t$ are consistent with a perfect capital reallocation through the credit market ($\omega_t = 1$) in the general equilibrium —given the state of Nature $\{A_t, Z_t, K_t\}$.

Corollary 2 indicates that crises may emerge through a fall in aggregate output (the "Y–channel"), a rise in retailers' markup (the "$\mathscr{M}$–channel"), or excess capital accumulation (the "K–channel"). Given a (predetermined) capital stock K_t, a crisis is more likely to break out following a shock that lowers output or increases the markup, as productive firms' return on capital r_t^k may fall below the crisis threshold $\bar{r}^k$ in either case. Such a shock does not need to be large to trigger a crisis, if the economy has accumulated a large enough capital stock. Indeed, when K_t is high, all else equal, productive firms' return on capital tends to be relatively low and the credit market is fragile. As we show in the next section, the capital stock may be especially high towards the end of an unusually long economic boom. In this case, even a modest drop in Y_t or rise in $\mathscr{M}_t$ may trigger a crisis.

The upshot is that the central bank may affect the probability of a crisis both in the short and in the medium run. In the short run, it may do so through the effect of contemporaneous changes in its policy rate on output and inflation (the Y– and $\mathscr{M}$–channels). To see this, consider the effects of an unexpected rate hike. On impact, the hike reduces aggregate demand for final and intermediate goods, weighing on retailers' prices and costs. In the presence of price adjustment costs, prices fall more slowly than costs and retailers' markups rise. The concomitant fall in aggregate demand and rise in the markup, in turn, weighs on productive firms' return on capital, bringing the economy closer to a crisis. In the medium run, in contrast, monetary policy affects financial stability through its impact on household saving behavior and capital accumulation (the K–channel). For example, a central bank that commits itself to systematically and forcefully responding to fluctuations in output will —all else equal— tend to lower the need for precautionary savings and to slow down capital accumulation during booms, thus enhancing credit market resilience.

5 Anatomy of a Financial Crisis

This section describes the average dynamics around financial crises under a plausible parametrization of the model. It also briefly links the model–based crisis dynamics with those observed in the data, with a focus on the role of monetary policy.

5.1 Parametrization of the Model

All the relevant equations of the model are reported in Table B.1 in Appendix B. Compared to the baseline NK model, our model has two additional parameters: the share of unproductive firms μ in the economy and creditors' recovery rate θ. Parameter μ directly affects the cost of financial crises in terms of productivity and output loss (see relation (33)). The debt recovery rate parameter θ governs the degree of moral hazard and, therefore, the incidence of financial crises. The definition of $\bar{r}^k$ in condition (29) shows that, given μ, the lower θ, the higher the crisis threshold $\bar{r}^k$, and the more frequent financial crises are.

Table 1: Parametrization

Parameter	Target	Value
Preferences		
β	4% annual real interest rate	0.989
σ	Logarithmic utility on consumption	1
φ	Frisch elasticity equals 2	0.50
χ	Steady state hours equal 1	0.81
Technology and price setting		
α	64% labor share	0.36
δ	6% annual capital depreciation rate	0.015
ϱ	Same slope of the Phillips curve as in Calvo price setting	58.22
ϵ	20% markup rate	6
Aggregate TFP (supply) shocks		
ρ_a	Standard persistence	0.95
σ_a	Volatility of inflation and output	0.007
Aggregate risk–premium (demand) shocks		
ρ_z	Standard persistence	0.95
σ_z	Volatility of inflation and output	0.001
Interest rate rule TR93 (Taylor (1993))		
ϕ_π	Response to inflation	1.5
ϕ_y	Response to output	0.125
Financial Frictions		
μ	Productivity falls by 1.8% due to financial frictions during a crisis	0.05
θ	The economy spends 10% of the time in a crisis	0.527

We parameterize our model based on quarterly data under Taylor (1993)'s original monetary policy rule (*i.e.* with $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$). The non–financial parameters are the same as in the baseline NK model and take the usual values (see Table 1). The utility function is logarithmic with respect to consumption ($\sigma = 1$). The labor disutility parameters are set to $\chi = 0.81$ and $\varphi = 0.5$, which normalize hours to one in the deterministic steady state and imply a Frisch labor supply elasticity of 2, within the range of calibrated values commonly used in the literature. We set the discount factor to $\beta = 0.989$, which corresponds to an annualized average return on financial assets of about 4%. The elasticity of substitution between intermediate goods, ϵ, is set to 6, which generates a markup of 20% in the deterministic steady state. Given this, we set the capital elasticity parameter α to 0.36 to obtain a labor income share of 64%. We assume that capital depreciates by 6% per year ($\delta = 0.015$). We set the price adjustment cost parameter to $\varrho = 58.2$, so that the model generates the same slope of the Phillips curve as in a

Calvo pricing model with an average duration of prices of 4 quarters.

The persistence of the technology and demand shocks is set to $\rho_a = \rho_z = 0.95$. The standard deviations of these two shocks are set *jointly* with parameter θ so that, in the stochastic steady state, the model replicates the volatility of the Hodrick–Prescott quarterly cyclical components of core inflation and output in the post–WW2 period and the economy spends 10% of the time in a crisis.[21] We obtain $\sigma_a = 0.007$, $\sigma_z = 0.001$, and $\theta = 0.527$. Note that, even though we do not set θ to match creditors' recovery rate *per se*, this value for θ is in line with the observed average recovery rate for the United States as well as with that considered in the literature.[22]

Finally, given $\alpha = 0.36$, we set $\mu = 0.05$ so that capital misallocation induces a further 1.8% ($= 1 - (1 - 0.05)^{0.36}$) fall in aggregate productivity during a financial crisis, all else equal.[23] This (momentary) productivity loss comes *on top of* that due to the adverse TFP shock that may trigger the crisis.

5.2 Simulated Dynamics Around Financial Crises

The aim of this section is to describe the dynamics of our model around financial crises. We compute these dynamics in three steps. First, we solve our non–linear model numerically using a global solution method —see details in Section I of the Online Appendix. Second, we feed the model with supply and demand shocks that follow standard AR(1) processes (see Table 1), simulate the model over 10,000,000 periods and "let the model speak". We thus obtain simulated time series of the endogenous variables and the underlying shocks. Third, we identify the starting dates of financial crises and compute the averages of macro–financial variables in the 24 quarters around these dates. To filter out potential noise arising from the aftermath of past crises, we restrict attention to "new" crises, defined as crises that occur after at least 24 quarters of normal times. Importantly, we do not impose any particular sequence of shocks to generate financial crises. Rather, given the model's internal dynamics, we identify *a posteriori* both the type of shocks —supply versus demand— and the sequence of shocks that give rise to financial crises.

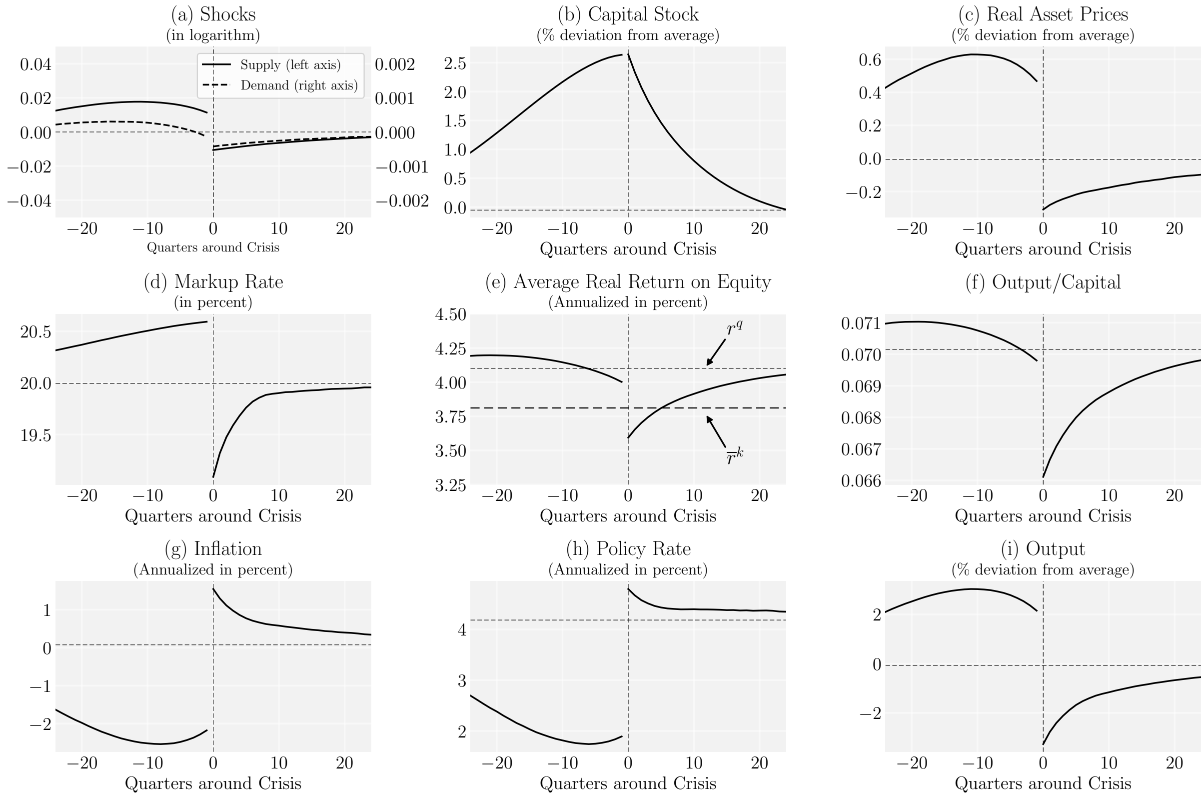
The average crisis dynamics, reported in Figure 4, are the outcome of both the two exogenous non–financial shocks (panel (a)) and the endogenous response of the economy to these shocks

[21] Romer and Romer (2017) and Romer and Romer (2019) construct a semiannual financial distress index for 31 OECD countries and rank the degree of credit disruption from 0 ("no stress") to 14 ("extreme crisis"). Using their index and definition of a financial crisis (*i.e.* a situation where the financial distress index is above or equal to 4), we compute the percentage of the time these countries spent in crises over the period 1980–2017, and obtain 10.57%. One advantage of the Romer and Romer (2017) index is that it allows us to parameterize the model to match the time spent in a crisis —instead of the probability of a crisis. More than 60% of Romer and Romer (2019)'s crises are also in JST and feature a financial panic (as defined in Baron et al. (2020)).

[22] In the case of the United States, Jankowitsch et al. (2014) document that the recovery rate on defaulted bonds hovered between 40% and 70% of bonds' face value over the period 2004–08 (Figure 4, p. 167). Longstaff et al. (2005)'s model–based analysis of market valuation of corporate debt assumes a constant recovery rate of 50% (p. 2225).

[23] While there is general agreement that financial frictions impair the reallocation of capital across firms —notably during financial crises, the resultant aggregate productivity loss is hard to measure. Estimates vary with the type of data, period, and methodology, *e.g.*: 0.8% in Oulton and Sebastiá-Barriel (2016) using aggregate data on 61 countries over the period 1950–2010 (p. 1); 1.76% in Gilchrist et al. (2013) for a sample of US manufacturing firms over the period 1985–2010 (Table 4, column (1), row $\alpha = 0.36$); 2.39% in Duval et al. (2019) using firm–level data for 11 advanced economies around the GFC (p. 487). We opt for a median value of 1.8%.

Figure 4: Simulated Dynamics Around Crises



Notes: Average dynamics of the economy around the beginning of a crisis (in quarter 0) in the stochastic steady state of the TR93 economy. To filter out the potential noise due to the aftershocks of past crises, we only report averages for new crises, *i.e.* crises that follow at least 24 quarters of normal times. The horizontal dotted lines correspond to the average values in the stochastic steady state. In panel (e), the upper horizontal dashed line corresponds to the deterministic steady state value r^q of the rate of return on equity, while the lower horizontal dashed line corresponds to the crisis threshold $\bar{r}^k$ as defined in relation (29). Since the capital stock is financed externally through equity issuance, its dynamics (panel (b)) correspond to those of equity funding (not reported). In Section L.2.1 of the Online Appendix, we show that, to finance their startup capital, firms are indifferent between issuing equity or risk–free debt, provided that they can issue such debt. In that case, the dynamics of the capital stock in panel (b) can also be interpreted as those of risk–free debt. The asset price reported in panel (c) corresponds to the real price of an asset that returns one unit of consumption good next period, *i.e.* $\beta\mathbb{E}_t\left[(C_{t+1}/C_t)^{-\sigma}\right]$ (see, *e.g.* Cochrane (2001)). In the stochastic steady state, the *average* crisis dynamics are essentially the same as the *median* ones for all variables except the aggregate shocks (panel (a)), whose values we discretized for the purpose of the numerical solution of the model (see Section I of the Online Appendix). The median dynamics of the shocks are nothing but a "stepwise version" of the average ones. Without loss of generality, for each variable, we report the dynamics in levels or in percentage deviation from average —whichever is more appropriate.

(other panels). The results suggest that these dynamics can be broadly decomposed into three phases: a boom, a slowdown, and a bust.

The Boom. The average crisis dynamics begin with a protracted sequence of small positive productivity and demand shocks, occurring 8 to 24 quarters before the start of the crisis (Figure 4, panel (a)). These positive shocks give rise to an economic boom (panel (i)), an investment and asset price boom (panels (b) and (c)) as well as high rates of return, as measured by the return on equity (panel (e)) and output–to–capital ratio (panel (f)), which are both above their respective average steady state values.[24]

Since positive productivity and demand shocks have opposite effects on prices, the dynamics of inflation indicate which shock dominates.[25] The prolonged fall in inflation and rise in markups (panels (g) and (d)) suggests that, on balance, the boom is mainly driven by the productivity shocks. Under TR93 (our baseline), the persistent disinflationary pressures induce the central bank to cut its policy rate and to keep it low for long (panel (h)). As a result, the boom that precedes the average crisis is characterized by both low inflation and monetary easing. Monetary easing, in turn, further boosts investment and capital accumulation.

The Slowdown. During the eight quarters that precede the crisis, productivity gains begin to subside and output falls toward its steady state (panels (a) and (i)). But as long as productivity remains above its steady state, households continue to accumulate savings and capital (panel (b)). Meanwhile, inflation picks up but remains below its steady state (panel (g)), and markups continue to rise (panel (d)). The combination of lower productivity, higher markups and a large capital stock weighs on firms' real equity returns, which fall below their steady state during the slowdown (panels (e) and (f)).

The Bust. The fall in capital/equity returns below their steady state value marks the entry of the economy into a region of financial fragility. Lower capital returns entail a lower credit–market equilibrium rate, which entices unproductive firms to search for yield and stokes lenders' fear of default. The credit market eventually breaks down as a relatively modest adverse productivity shock (and the endogenous response of the economy thereto) pulls capital/equity returns further down and below the crisis threshold (panels (a), (e) and (f)). The average crisis is characterized by a severe recession (panel (i)) and asset–price correction (panel (c)). On average, output falls by 6% during a crisis (Table 2, row (1), column "Output Loss").

Role of Monetary Policy and Link to Facts. Despite its stylized nature, our model does a fair job in capturing and articulating the salient facts about post–WW2 financial crises, as identified in the literature and summarized in Figure 1 (compare Figures 1 and 4). It also suggests that monetary policy plays an important role in the unfolding of events that precede crises. Figure 5 depicts the transmission chain at play.[26] At first, a protracted rise in productivity

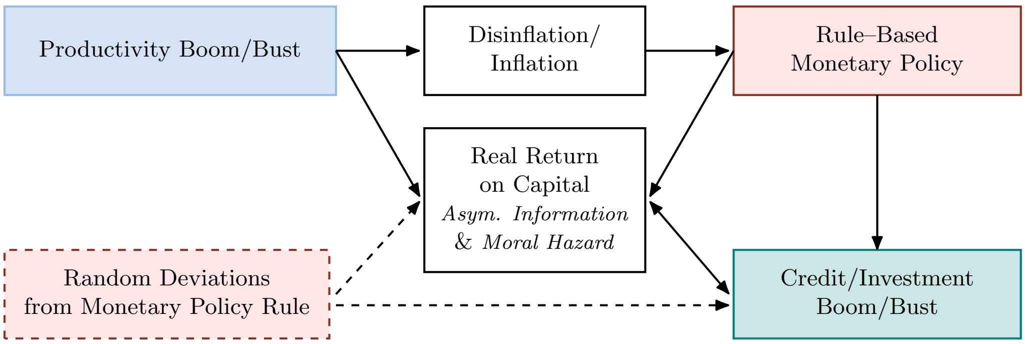
[24]In the baseline version of the model investment is entirely financed externally through equity issuance, implying that the investment boom goes hand in hand with an equity issuance boom. As noted in Section L.2 of the Online Appendix, however, our model is isomorphic to an alternative model where firms finance their capital stock with risk–free debt. In that case, the investment boom in panel (b) can be interpreted as a credit boom.

[25]In Section D of the Online Appendix, we report the dynamics of crises in a version of the model with either supply or demand shocks —as opposed to both. The comparison of these dynamics makes clear that demand–driven booms are inflationary while supply–driven ones are disinflationary.

[26]This transmission chain applies to the average crisis, which tends to follow a productivity–driven boom, as Figure 4 suggests. It does not apply to the few crises that follow demand–driven booms (see Figure D.2 in Section D of the Online Appendix) or to those that do not follow a boom. Further note that the effects of monetary policy on financial stability depend on the type of shocks hitting the economy. When demand shocks prevail, capital accumulation goes hand in hand with inflationary (rather than disinflationary) pressures. In that case, raising the policy rate to tame inflation helps to slow down capital accumulation and to reduce the risk of financial stress in the medium run in our model, as shown empirically in Boissay et al. (2025).

leads to faster capital accumulation and to a disinflationary boom.[27] The central bank responds to disinflationary pressures by keeping its policy rate low for long (Figures 1 and 4; panels (g) and (h)), which boosts aggregate demand, raises capital returns, and further stimulates capital accumulation (Figures 1 and 4; panel (b)). Throughout the boom, capital accumulation gradually reduces firms' real return on capital (Figures 1 and 4; panels (e) and (f)) thereby slowing investment and growth and increasing credit markets' vulnerability to adverse shocks. Eventually, a decline in productivity generates inflationary pressures, leading the central bank to raise its policy rate (Figures 1 and 4; panels (g) and (h)). As monetary policy tightening dampens aggregate demand, the real return on capital declines, eventually precipitating the bust. At this stage, the rate hike operates more as a catalyst than as the root cause of the crisis, insofar as the same tightening may not have triggered a crisis had the policy rate not remained low for long in the first place.

Figure 5: Boom–Slowdown–Bust Episodes: the Role of Productivity and Monetary Policy



Notes: This diagram summarizes the interactions between the main macro–financial factors (credit/investment, productivity, monetary policy) that underpin the average crisis dynamics in the model (Figure 4).

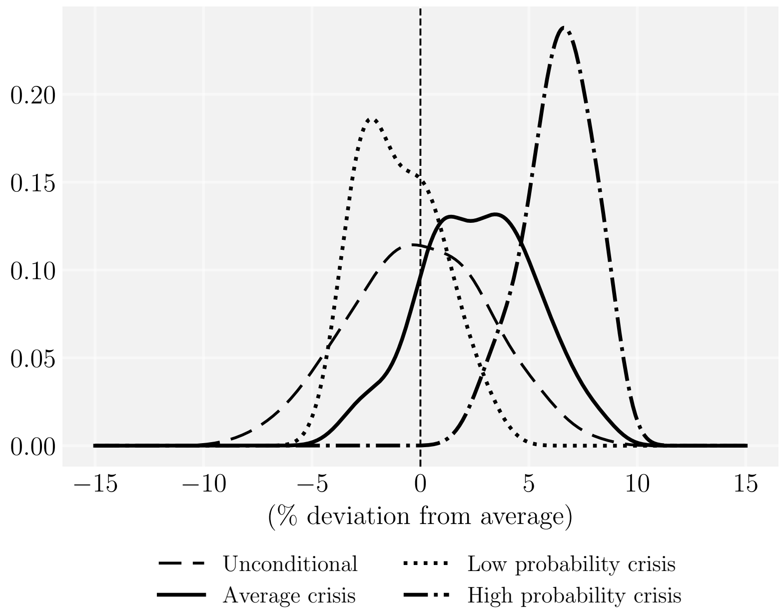
Throughout the boom–slowdown–bust episode, the monetary policy rate follows a U–shaped path that goes hand in hand with a U–shaped path of inflation (Figures 1 and 4; panels (g) and (h)), as the central bank aims to stabilize inflation under TR93. This pattern is consistent with the recent empirical finding by Jiménez et al. (forthcoming) as well as with the narratives of prominent historical crises such as the GFC in the United States and the Swedish and Japanese crises in the early 1990s, which all followed disinflationary booms.

Variety of Crises and Crisis Probability. The average crisis dynamics described in Figure 4 mask the variety of crises in terms of their origins and underlying transmission channels —the Y–$\mathscr{M}$–K– channels. To illustrate this variety, we report in Figure 6 the ergodic distributions

[27] Other papers have explained the disinflationary pressures observed in the run–up to financial crises in different ways. For instance, Brunnermeier and Julliard (2008) and Piazzesi and Schneider (2008) argue that the disinflation in the first half of the 2000s in the United States may have fueled the credit and asset price boom that preceded the GFC, because of money or inflation illusion. Ikeda (2022), in contrast, argues that it is a sentiment–driven asset price boom that may have fueled disinflation by boosting firms' collateral value and lowering their funding and production costs. In the same vein, Christiano et al. (2010) show that overoptimistic beliefs about future productivity may lead to boom–bust dynamics characterized by low–for–long inflation and policy rates, followed (as in our model) by a rise in inflation and interest rates.

of the capital stock in the quarter before all crises (solid line); before crises whose *ex ante* (*i.e.* one–quarter ahead) probability was in the bottom decile of its ergodic distribution (dotted line); and before crises whose *ex ante* probability was in the top decile of its ergodic distribution (dash–dotted line). On average, crises occur when the capital stock is about 2.5% above its average level (solid line). However, high–probability crises are associated with a much larger investment boom, with the capital stock more than 5% above average (pointing to the K–channel as the main driver) whereas low–probability crises occur when the capital stock is *below* average. The latter crises ought to be due to relatively large —and therefore rare and unexpected— adverse shocks that lower productive firms' return on capital through the Y–$\mathscr{M}$–channels.

Figure 6: Capital Stock Distribution Across Crises



Notes: Kernel density estimates of the ergodic distributions of the capital stock in the TR93 economy, unconditional (dashed line) and conditional on being in a crisis next quarter (solid lines). A crisis has a low (high) *ex ante* probability if its one–quarter ahead probability in the quarter before it broke out was in the bottom (top) decile of its unconditional distribution.

These findings highlight two noticeable features of financial crises in our model. For one, most but not all crises follow an investment boom.[28] For another, those that follow an investment boom tend to have a higher *ex ante* probability and, in that sense, to be more "predictable" than other crises. In the stochastic steady state, up to one third of the crises have an *ex ante* (one–quarter ahead) probability above 95%.

5.3 Crisis Anticipations and Externalities

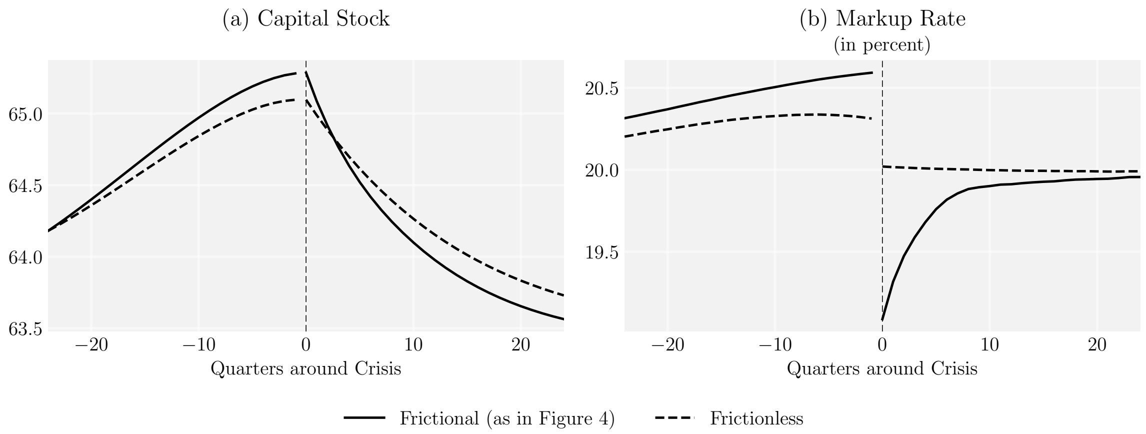
The latter result prompts the question of why financial crises break out even though agents can anticipate them. The reason is that neither households nor retailers internalize the effects of their individual choices on financial fragility and that, somewhat paradoxically, their anticipation

[28]This result is consistent with Greenwood et al. (2022), who document that 21.1% of the financial crises do not follow a credit and asset price boom or (in their language) a "R–zone" (Table X, panel A, column "both").

of a crisis induces them to precipitate (rather than avert) it.

When a crisis is looming, households seek to hedge against the future recession and smooth their consumption by accumulating precautionary savings, which contributes to an even larger capital stock. Boissay et al. (2016) refer to this phenomenon as a "savings glut" externality.[29]

Figure 7: Crisis Anticipations, Saving/Capital Glut and Markup Externalities



Notes: Crisis dynamics in a TR93 economy with a frictional *versus* frictionless credit market. Frictional case (solid line): same average dynamics as in Figure 4. Frictionless case (dashed line): counterfactual average dynamics, when the simulations start with the same capital stock in quarter −24 and the economy is fed with the same aggregate shocks as the frictional credit market economy. To illustrate that the simulations start with the same capital stock, the dynamics are reported in levels.

Similar financial externalities stem from retailers. All else equal, the collapse of the credit market during a crisis induces a fall in aggregate productivity (recall relations (27) and (33)), and hence generates weaker disinflationary (or stronger inflationary) pressures than in an economy with a frictionless credit market. To smooth their price adjustment costs over time, retailers lower their prices by less (or increase them by more) ahead of a crisis, thus raising their markup above the level that would otherwise prevail absent financial frictions. As higher markups reduce firms' return on capital, retailers' response to financial fragility makes the credit market more fragile.

Figure 7 shows that these externalities are underpinned by agents' anticipation of a crisis. This figure compares the dynamics of capital and markups around a crisis with their dynamics in a counterfactual economy without financial frictions that is fed with the same shocks. Our focus is on *the run–up phase* to the average financial crisis, *i.e.* on quarters −24 to −1. Since the credit market works equally well in the two economies during that period, the difference between the solid and the dashed lines reflects agents' anticipation of a crisis and the response thereto. Both the capital stock and markup are higher when households and retailers anticipate a crisis, revealing the presence of the savings glut and markup externalities. These externalities have implications for the calibration of monetary policy, which we study next.

[29]Such an externality leads to excess savings and capital accumulation. In that respect, our model differs from other models that emphasize the role of pecuniary externalities in the build–up of excessive borrowing and leverage (*e.g.* Lorenzoni (2008), Bianchi (2011)).

6 The "Divine Coincidence" Revisited

In the absence of financial frictions, SIT simultaneously eliminates inefficient fluctuations in prices and in the output gap and achieves the first–best allocation —the so–called "divine coincidence" (Blanchard and Galí (2007)), as shown in Table 2 (row (6), column "Frictionless"). In the presence of financial frictions, in contrast, SIT does not deliver the first–best allocation. In our model, the welfare loss under SIT is strictly positive and amounts to 0.23% in terms of consumption equivalent variation (Table 2, row (6), column "Welfare Loss").

Can (and should) central banks prevent financial crises by tolerating higher inflation volatility? To answer this question, we study the trade–off between price and financial stability and compare welfare under SIT *versus* alternative policy rules. We consider three types of rules: standard Taylor rules, Taylor–type rules augmented with the household's return on equity, and regime–contingent rules. Throughout this section, we focus on *systematic* monetary policy and abstract from unexpected random deviations from the preannounced policy rule (which we will study in Section 7).

6.1 Price *versus* Financial Stability Trade–off

Comparing the effects of various Taylor–type rules reveals a trade–off between price and financial stability. We find that the central bank can reduce the incidence and severity of crises by deviating from price stability and responding more strongly to output in addition to inflation. More precisely, Table 2 shows that, all else equal, raising ϕ_y from 0.125 to 0.375 in the Taylor–type rule (1) reduces the percentage of the time spent in a crisis from 10% to 3.7% (Table 2, rows (1) *versus* (3), column "Time in Crisis") as well as the output loss due to a crisis from 6% to 3.9% (column "Output Loss"). However, these financial stability gains come at the cost of twice as much inflation volatility (2pp compared to 1pp, column "Std(π_t)").

Price instability can also contribute to financial fragility through markups ($\mathscr{M}$–channel; see Section 4). All else equal, raising ϕ_π from 1.5 to 2.5 in the Taylor–type rule (1) reduces both the volatility of inflation from 1pp to 0.4pp (rows (1) *versus* (5), column "Std(π_t)") and the time spent in crisis from 10% to 9.8% (column "Time in Crisis"). However, improvements in financial stability *via* the $\mathscr{M}$–channel are limited, with a hard lower bound of crisis incidence at 9.6% under SIT (row (6)). Further reducing the time spent in crisis requires departing from SIT at the cost of inflation volatility (rows (2) and (3)). Hence, the central bank faces a trade–off between price and financial stability in our model.

Which leg of the trade–off dominates in terms of welfare is a quantitative question. Given our parametrization, we find that, on balance, the welfare loss due to price instability more than offsets the gain from enhanced financial stability under *standard* Taylor–type rules (rows (2)–(5) *versus* row (6), column "Welfare Loss"). Hence, even though it is associated with a strictly positive (0.23%) welfare loss due to a relatively high incidence and severity of financial crises, SIT improves welfare relative to standard Taylor–type rules.

Table 2: Economic Performance and Welfare Under Alternative Policy Rules

	Rule	Rule parameters ϕ_π	ϕ_y	ϕ_r	Model with Financial Frictions: Time in crisis or stress (in %)	Length (quarters)	Output Loss (in %)	Std(π_t) (in pp)	Welfare Loss (in %)	Frictionless: Welfare Loss (in %)
					Taylor–type Rules					
(1)	TR93	1.5	0.125	–	[10.0]	4.7	6.0	1.0	0.59	0.33
(2)	TR	1.5	0.250	–	7.3	4.0	5.0	1.5	1.04	0.83
(3)	TR	1.5	0.375	–	3.7	2.8	3.9	2.0	1.87	1.46
(4)	TR	2.0	0.125	–	9.9	5.0	6.6	0.5	0.35	0.10
(5)	TR	2.5	0.125	–	9.8	5.0	6.8	0.4	0.29	0.05
					Strict Inflation Targeting					
(6)	SIT	$+\infty$	–	–	9.6	5.1	7.5	0.0	0.23	0.00
					Augmented Taylor–type Rules					
(7)	A-TR93	1.5	0.125	5.0	5.3	3.7	5.1	1.0	0.48	–
(8)	A-TR	5.0	0.125	5.0	9.1	4.9	6.8	0.2	0.23	–
(9)	A-TR	10.0	0.125	25.0	8.4	4.8	6.6	0.2	0.22	–
					Backstop Rules					
(10)	B-TR93	1.5	0.125	–	16.2	–	–	0.9	0.33	–
(11)	B-SIT	$+\infty$	–	–	18.8	–	–	0.5	0.10	–

Notes: TR93: baseline Taylor rule as in Taylor (1993) — quarterly version with $\phi_\pi = 1.5$, $\phi_y = 0.125$, and $\phi_r = 0$; TR: Taylor–type rule; SIT: strict inflation targeting rule; A–TR(93): (baseline) Taylor–type rule augmented with the "yield gap"; B–TR93(SIT): baseline Taylor–type (strict inflation targeting) rule in normal times only and full backstop of the credit market by the central bank in stress times. All statistics correspond to the stochastic steady state. "Time in Crisis/Stress" is the percentage of the time that the economy spends in a crisis (rows (1)–(9)) or in financial stress (rows (10)–(11)). "Length" is the average duration of a crisis/stress period (in quarters). "Output Loss" is the percentage fall in output from one quarter before the crisis until the trough of the crisis (in %). "Std(π_t)" is the standard deviation of inflation (in percentage points). "Welfare Loss" is the loss of welfare relative to the first–best economy, expressed in terms of consumption equivalent variation (in percent), and corresponds to the percentage of permanent consumption the household would have to forgo in the first–best economy to reach the same level of welfare as in our economy with nominal and financial frictions. In the case of the frictionless credit market economy (column "Frictionless"), the SIT economy reaches the first best and there is no welfare loss in this case. In the case of the frictional credit market and the TR93 rule, the economy spends by construction 10% of the time in a crisis (square brackets; see Section 5.1).

Next, we ask whether following alternative —more "informed"— Taylor–type rules could improve welfare. A candidate rule is one whereby the central bank responds positively to the household's return on equity r_t^q (on top of inflation and output) and, more particularly, to a "yield gap" defined as the difference between r_t^q and its deterministic steady state value r^q. Indeed, Figure 4 (panel (e)) suggests that a positive yield gap ($r_t^q > r^q$) is associated with a booming economy while a negative yield gap tends to precede crises. Accordingly, we consider the following *augmented* Taylor rule ("A–TR" rules in Table 2),

$$1 + i_t = \frac{1}{\beta}(1+\pi_t)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_y}\left(\frac{1+r_t^q}{1+r^q}\right)^{\phi_r} \tag{37}$$

with $\phi_r > 0$. There are good reasons why such a rule may improve welfare. On the one hand, and all else equal, it requires setting the policy rate above that of the corresponding standard Taylor rule during economic booms, when the yield gap is positive ($r_t^q > r^q$). In turn, a higher policy rate helps to slow down capital accumulation and keep financial imbalances from building up during such booms. On the other hand, the augmented Taylor rule also requires the central bank to set the policy rate below that of a standard rule when the economy approaches a crisis and the yield gap turns negative ($r_t^q < r^q$). At that point, lowering the policy rate may help to boost aggregate demand and steer the economy away from the financial fragility region.

Table 2 shows that responding to the yield gap fosters financial stability and increases welfare compared to standard Taylor–type rules. For example, the economy spends only 5.3% of the time in a crisis under the augmented TR93 rule (A–TR93) with $\phi_r = 5$, against 10% under TR93 (row (7) *versus* (1), column "Time in Crisis"). Setting $\phi_r > 0$ does not materially affect inflation volatility compared to TR93, implying a positive net effect on welfare: the welfare loss falls from 0.59% under TR93 to 0.48% under the A–TR93 rule (row (1) *versus* (7), column "Welfare Loss"). In turn, responding more aggressively to inflation helps to lower the overall welfare loss to 0.23%, *i.e.* to the same level as under SIT (row (6) *versus* (8)). After trying several values for ϕ_π, ϕ_y, and ϕ_r, we could reduce the welfare loss further to 0.22% (row (9)) but not much beyond that. This result suggests that, compared to SIT, the cost of experiencing higher inflation volatility in normal times under augmented Taylor rules broadly balances the benefit of experiencing fewer financial crises.

6.2 "Backstop" Rules

We now study regime–contingent monetary policy rules, whereby the central bank commits itself to following TR93 or SIT in normal times *and* to doing whatever is needed —and therefore exceptionally deviating from these rules— to forestall a crisis whenever necessary. In those instances, we assume that the central bank deviates "just enough" to avert the crisis, *i.e.* sets its policy rate so that $r_t^k = \bar{r}^k$ (see Proposition 2).[30] We call such a rule a "backstop rule".

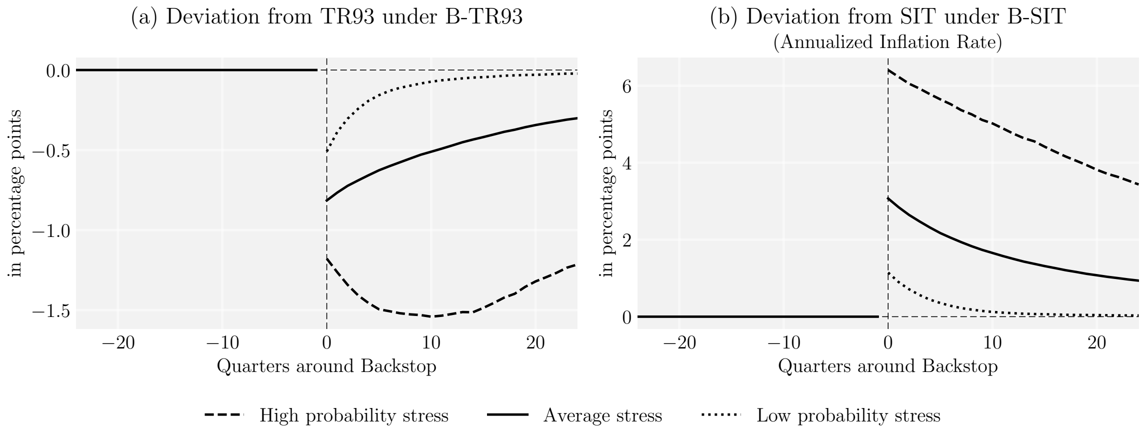
There are two good reasons for considering backstop rules. The first is conceptual. As a financial crisis corresponds to a regime shift, a rule that is followed in —and designed for— normal times is unlikely to be appropriate during periods of financial stress. In effect, regime switches call for a regime–contingent strategy. Our contention is that, by giving the central bank more flexibility, such a strategy can alleviate the trade–off between price and financial stability discussed in the previous section. The second reason is practical: our backstop rule speaks to the "backstop principle" that most central banks in advanced economies have *de facto* been following since the GFC and that involves deviating from conventional ("normal times") monetary policy when necessary to restore credit market functionality. Our analysis can therefore be seen as an attempt to assess the benefits of post–GFC monetary policy strategies.

We show below that backstop rules can significantly improve welfare compared to both SIT and Taylor–type rules. We start by reporting in Figure 8 the average systematic deviations from TR93 and SIT that are needed during stress times to ward off crises (solid lines) and refer to these backstop policies as B–TR93 (panel (a)) and B–SIT (panel (b)), respectively. The

[30] In the case of a Taylor–type rule $1 + i_t = (1 + \pi_t)^{1.5} (Y_t/Y)^{0.125} \varsigma_t/\beta$, for example, this consists in setting the term ς_t equal to 1 if $r_t^k \geq \bar{r}^k$ and such that $r_t^k = \bar{r}^k$ whenever (and only then) r_t^k would be otherwise lower than $\bar{r}^k$. Likewise, under SIT, the central bank tolerates just enough deviations from the inflation target so that $r_t^k = \bar{r}^k$.

deviations are reported in terms of the policy rate for B–TR93 and the annualized inflation rate for B–SIT.

Figure 8: Backstop Necessary to Stave off a Crisis and Normalization Path



Notes: Average deviations from the normal–times policy rule that the central bank must commit to in order to forestall a financial crisis, and the normalization path (after quarter 0). Panel (a): deviation of monetary policy rate from TR93, in percentage points. Such a deviation of the policy rate is akin to what Akinci et al. (2020) call "$R^{\star\star}$". Panel (b): deviation of the inflation target from zero, in percentage points, when the central bank follows SIT. For the purpose of the exercise, financial stress is defined as a situation where there would have been a crisis absent the monetary policy backstop. A stress episode has a low (high) *ex ante* probability if its one–quarter ahead probability in the quarter that precedes it is in the bottom (top) decile of its unconditional distribution.

Figure 8 indicates that the central bank must loosen its policy compared to normal times, *i.e.* cut the policy rate by almost 1 percentage point below TR93 or temporarily tolerate an inflation rate 3 percentage points higher under SIT.[31] It also shows that the backstop policy must be unwound gradually, reflecting the time it takes for the underlying financial vulnerabilities to dissipate. The appropriate normalization path is narrow: tightening monetary policy more slowly would lead to unnecessarily high inflation and costs due to nominal rigidities; tightening more quickly would result in a financial crisis and a "hard landing".

One important determinant of the speed of normalization is the type of financial vulnerabilities that are being addressed. When the stress is due to an exogenous adverse shock ("Low probability stress"), the central bank can set its policy rate (roughly) in line with the TR93 rule as early as after 10 quarters (panel (a), dotted line). When it is due to an excessive investment boom ("High probability stress"), in contrast, the normalization takes much longer and is still far from over after 24 quarters (dashed line). The reason is intuitive. As the central bank intervenes to stem stress, it also slows down the adjustment that would be necessary to eliminate the savings glut that causes stress in the first place. As a result, monetary policy must remain accommodative for longer to prevent a crisis.[32]

Finally, we study the net welfare gain of following a backstop rule. The results are reported

[31]Throughout, we assume that the central bank is not constrained by a zero lower bound on the nominal policy rate. Conditional on being in stress times (when backstop policies are activated), the nominal policy rate is negative 6.8% of the time under B–TR93.

[32]In models where crises are driven by an exogenous financial shock to borrowers' collateral or net worth (*e.g.* Andrés et al. (2013), Manea (2020)), the optimal policy also consists in cutting policy rates in a crisis.

at the bottom of Table 2. Two findings stand out.

First, backstopping the economy unambiguously improves welfare. In the case of TR93, the welfare loss is reduced from 0.59% without a backstop to 0.33% with a backstop (row (1) *versus* (10), column "Welfare Loss"). The welfare loss is essentially the same in that case as in the economy with no financial frictions (row (1), column "Frictionless"). In the case of SIT, the welfare loss falls by more than half, from 0.23% without a backstop to 0.1% with a backstop (row (6) *versus* (11), column "Welfare Loss").

Second, the financial sector is *more* fragile when the central bank commits itself to backstopping the economy. Under B–SIT, for instance, the central bank has to backstop the economy and deviate from its normal–times policy rule 18.8% of the time, whereas under SIT the economy would spend only 9.6% of the time in a crisis (row (11) *versus* (6), column "Time in Crisis/Stress"). The reason is that, as they forestall financial crises, backstop policies also delay the reduction in the capital stock that would be necessary to sustain high capital returns and deter search–for–yield behavior. On average, the capital stock is therefore higher under B–SIT than under SIT, thus increasing the fragility of the credit market.[33]

7 Deviations from the Taylor Rule and Financial Instability

To what extent may monetary policy itself unintentionally breed financial vulnerabilities? One way to answer this question is to study the potentially detrimental effects of *random* deviations from the policy rule —or "policy surprises".[34] To pinpoint these effects, we consider a TR93 economy that experiences monetary policy shocks and in which these shocks are the only source of aggregate uncertainty —*i.e.* we discard supply and demand shocks. More specifically, we consider the monetary policy rule

$$1 + i_t = \frac{1}{\beta}(1 + \pi_t)^{1.5} \left(\frac{Y_t}{Y}\right)^{0.125} \varsigma_t$$

where the monetary policy shock ς_t follows an AR(1) process $\ln(\varsigma_t) = \rho_\varsigma \ln(\varsigma_{t-1}) + \epsilon_t^\varsigma$, with $\rho_\varsigma = 0.5$ and $\sigma_\varsigma = 0.0025$, as in Galí (2015). We study the dynamics of monetary policy shocks in the run–up to crises in this new environment.

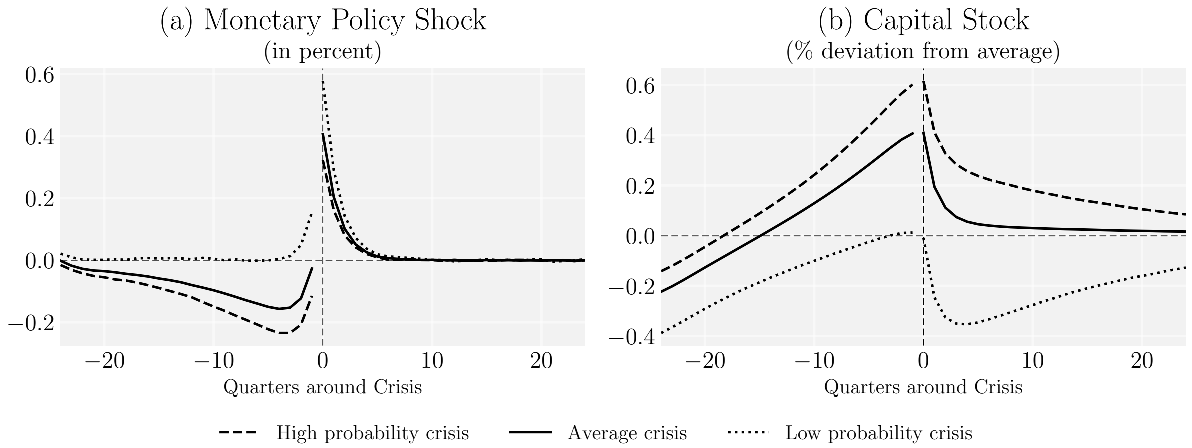
Figure 9 (panel (a)) shows that the average crisis breaks out after a long period of unexpected monetary easing as the central bank reverses course (solid line). Keeping monetary policy loose for long indeed stimulates capital accumulation (panel (b)), which in turn undermines the resilience of the credit market to shocks via the K–channel. The crisis is then triggered by three consecutive and unexpected monetary policy rate hikes toward the end of the boom. The comparison between crises with a high *ex ante* probability, shown by the dashed line, and those

[33] In Section H of the Online Appendix, we show that backstops, however, slow down the accumulation of precautionary savings and capital before financial stress episodes, because households factor them in and anticipate that the central bank will act to prevent stress from materializing into a crisis.

[34] This section revisits Taylor (2011)'s "Great Deviation" view of the GFC according to which unpredictably loose monetary policy may have exposed the economy to financial stability risks.

with a low *ex ante* probability, shown by the dotted line, further indicates that longer periods of loose monetary policy require smaller rate hikes to trigger a crisis.

Figure 9: Loose Monetary Policy for Too Long May Lead to a Crisis



Notes: Average random deviations from TR93 (panel (a)) and the capital stock dynamics (panel (b)) around a crisis (quarter 0) in an economy with only monetary policy shocks. A crisis has a low (high) *ex ante* probability if its one–quarter ahead probability in quarter −1 was in the bottom (top) decile of its unconditional distribution.

These findings are consistent with recent empirical evidence that a protracted period of unanticipated loose monetary policy followed by rapid monetary tightening is conducive to financial instability (Schularick et al. (2021), Grimm et al. (2023), Jiménez et al. (forthcoming)). More generally, our analysis highlights that random deviations from the monetary policy rule are themselves a potential source of financial instability (Figure 5, dashed arrows).

8 Concluding Remarks

We have proposed an extension of the textbook NK model, enriching it along several dimensions: capital accumulation, heterogeneous firms, a credit market that permits the efficient reallocation of capital across firms, and frictions in that market. Without financial frictions, the equilibrium outcome coincides with that of the standard representative–firm model. With financial frictions, by contrast, a borrowing constraint may prevent the full reallocation of capital toward the most efficient firms. When the return on capital falls (*e.g.* due to a protracted investment boom), borrowers have stronger incentives to invest in alternative (privately beneficial) projects, stoking lenders' fears of default and possibly causing a financial crisis. In such an environment, monetary policy affects the probability of a crisis in the short run —through its usual effects on aggregate demand— and in the medium run —through its effect on capital accumulation.

We use the model to conduct several monetary policy experiments. We show that a policy that consists of rapidly tightening monetary policy after having kept it loose for long can lead to financial crises. We also show that a central bank can increase welfare by providing backstops, *i.e.* committing to doing whatever is needed whenever necessary to forestall a crisis. Once backstops are activated, the speed at which monetary policy can be normalized depends on the source of financial vulnerabilities, *e.g.* a boom or a large adverse shock.

Our model is a first step toward more complex models that would feature a richer set of financial frictions, amplification mechanisms, and policies. In this paper, we purposely left out two potential ingredients that, while relevant, would not have added much insight into the role of monetary policy *in the genesis of financial crises*. Since we were interested in the effects of *monetary* policy, we abstracted from other policies, such as macro–prudential or fiscal policies. Our intention was not to argue that these policies are not effective or should not be used as a first line of defense against the build–up of financial vulnerabilities. Rather, it was to better understand how monetary policy can, by itself, create or mitigate financial stability risks. It would be interesting to introduce macro–prudential policies that help rein in the accumulation of capital during booms and to study the policy mix in that case. In the same vein, since our goal was to study *endogenous* financial crises and how financial vulnerabilities build up, we deliberately left out amplification mechanisms such as deleveraging and liquidity spirals due to collateral constraints. Such mechanisms would nonetheless be useful to further explain how even small financial shocks may end up having systemic effects. Finally, another extension of our model could consist of studying the effects of a zero lower bound on the nominal policy rate (ZLB) in a low–rate environment. The implications of such a constraint are not clear–cut: while a ZLB may constrain the use of backstop policies, it could also reduce the need for such interventions by capping rate cuts during disinflationary booms —thereby fostering financial stability in the medium run. These and other extensions are left for future research.

Data Availability Statement

The data and code underlying this article are available on Zenodo at **https://doi.org/10.5281/zenodo.16947470**.

References

O. Akinci, G. Benigno, M. Del Negro, and A. Queralto (2020): "The Financial (In)stability Real Interest Rate, $R^{\star\star}$," Federal Reserve Bank of New York Staff Report No. 946.

J. Andrés, O. Arce, and C. Thomas (2013): "Banking Competition, Collateral Constraints, and Optimal Monetary Policy," *Journal of Money, Credit and Banking*, 45(s2), 87–125.

C. Azariadis and B. Smith (1998): "Financial Intermediation and Regime Switching in Business Cycles," *American Economic Review*, 88(3), 516–536.

Bank for International Settlements (2022): "Market Dysfunction and Central Bank Tools," Insights from the Markets Committee Working Group on Market Dysfunction and Central Banks, https://www.bis.org/publ/mc_insights.htm.

M. Baron, E. Verner, and W. Xiong (2020): "Banking Crises Without Panics," *Quarterly Journal of Economics*, 136(1), 51–113.

B. Bernanke (2018): "The Real Effects of Disrupted Credit: Evidence from the Global Financial Crisis," *Brookings Papers on Economic Activity*, Fall, 251–322.

——— (2023): "Nobel Lecture: Banking, Credit, and Economic Fluctuations," *American Economic Review*, 113(5), 1143–69.

B. Bernanke and M. Gertler (1989): "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, 79, 14–31.

——— (1990): "Financial Fragility and Economic Performance," *The Quarterly Journal of Economics*, 105(1), 87–114.

B. Bernanke, M. Gertler, and S. Gilchrist (1996): "The Financial Accelerator and the Flight to Quality," *Review of Economics and Statistics*, 78(1), 1–15.

——— (1999): "The Financial Accelerator in a Quantitative Business Cycle Framework," *Handbook of Macroeconomics*, 1.

Javier Bianchi (2011): "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review*, 101(7), 3400–3426.

O. Blanchard and J. Galí (2007): "Real Wage Rigidities and the New Keynesian Model," *Journal of money, credit and banking*, 39, 35–65.

F. Boissay, F. Collard, C. Manea, and A. Shapiro (2025): "Monetary Tightening and Financial Stress during Supply- versus Demand-driven Inflation," *International Journal of Central Banking*, 21(2), 147–220.

F. Boissay, F. Collard, and F. Smets (2016): "Booms and Banking Crises," *Journal of Political Economy*, 124(2), 489–538.

P. Bordalo, N. Gennaioli, and A. Shleifer (2018): "Diagnostic Expectations and Credit Cycles," *The Journal of Finance*, 73(1), 199–227.

M. Bordo and D. Wheelock (2007): "Stock Market Booms and Monetary Policy in the Twentieth Century," *Federal Reserve Bank of Saint Louis Review*, 89(2), 91–122.

C. Borio (2006): "Monetary and Prudential Policies at a Crossroads? New Challenges in the New Century," BIS Working Papers 216.

C. Borio and P. Lowe (2002): "Asset Prices, Financial and Monetary Stability: Exploring the Nexus," BIS Working Papers 114.

M. Brunnermeier (2009): "Deciphering the Liquidity and Credit Crunch 2007–2008," *Journal of Economic Perspectives*, 23(1), 77–100.

M. Brunnermeier and C. Julliard (2008): "Money Illusion and Housing Frenzies," *The Review of Financial Studies*, 21(1), 135–180.

M. Brunnermeier and Y. Sannikov (2014): "A Macroeconomic Model with a Financial Sector," *American Economic Review*, 104(2), 379–421.

D. Cao and J-P. L'Huillier (2018): "Technological Revolutions and the Three Great Slumps: A Medium-run Analysis," *Journal of Monetary Economics*, 96, 93–108.

V. Cerra and S. Saxena (2008): "Growth Dynamics: The Myth of Economic Recovery," *American Economic Review*, 98(1), 439–57.

L. Christiano, C. Ilut, R. Motto, and M. Rostagno (2010): "Monetary Policy and Stock Market Booms," *Proceedings - Economic Policy Symposium - Jackson Hole*, pp. 85–145.

L. Christiano, R. Motto, and M. Rostagno (2014): "Risk Shocks," *American Economic Review*, 104(1), 27–65.

J. Cochrane (2001): *Asset Pricing*. Princeton University Press.

G. Corsetti, P. Pesenti, and N. Roubini (1999): "What Caused the Asian Currency and Financial crisis?," *Japan and the World Economy*, 11, 305–373.

T. Dang, G. Gorton, and B. Holmström (2020): "The Information View of Financial Crises," *Annual Review of Financial Economics*, 12(Volume 12, 2020), 39–65.

G. Dell'Ariccia, D. Igan, L. Laeven, and H. Tong (2016): "Credit Booms and Macrofinancial Stability," *Economic Policy*, 31(86), 299–355.

D. Duffie and F. Keane (2023): "Market-function Asset Purchases," Federal Reserve Bank of New York Staff Report No. 1054.

R. Duval, G. Hong, and Y. Timmer (2019): "Financial Frictions and the Great Productivity Slowdown," *Review of Financial Studies*, 33(2), 475–503.

P. Englund (1999): "The Swedish Banking Crisis: Roots and Consequences," *Oxford Review of Economic Policy*, 15(3), 80–97.

FCIC (2011): "The financial crisis inquiry report," Discussion paper, National Commission on the Causes of the Financial and Economic Crisis in the United States.

J. Fernald (2015): "Productivity and Potential Output Before, During, and After the Great Recession," *NBER Macroeconomics Annual*, 29, 1–51.

P. Fontanier (2025): "Optimal Policy for Behavioral Financial Crises," *Journal of Financial Economics*, 166, 104005.

L. Foster, C. Grim, and J. Haltiwanger (2016): "Reallocation in the Great Recession: Cleansing or not?," *Journal of Labor Economics*, 34, 296–331.

T. Fuerst (1995): "Monetary and Financial Interactions in the Business Cycle," *Journal of Money, Credit and Banking*, 27, 1321–1338.

J. Galí (2015): *Monetary policy, Inflation, and the Business Cycle: an Introduction to the New Keynesian Framework and its Applications*. Princeton University Press.

J. Galí (2018): "The State of New Keynesian Economics: A Partial Assessment," *Journal of Economic Perspectives*, 32(3), 87–112.

M. Garmaise (2015): "Borrower Misreporting and Loan Performance," *The Journal of Finance*, 70(1), 449–484.

M. Gertler (1992): "Financial Capacity and Output Fluctuations in an Economy with Multi-period Financial Relationships," *Review of Economic Studies*, 59, 455–472.

M. Gertler and P. Karadi (2011): "A Model of Unconventional Monetary Policy," *Journal of Monetary Economics*, 58(1), 17–34.

M. Gertler and N. Kiyotaki (2011): "Financial Intermediation and Credit Policy in Business Cycle Analysis," in *Handbook of Monetary Economics*, vol. 3A, chap. 11, pp. 547–599. Amsterdam: Elsevier Science.

——— (2015): "Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy," *American Economic Review*, 105(7), 2011–43.

M. Gertler, N. Kiyotaki, and A. Prestipino (2020): "A Macroeconomic Model with Financial Panics," *Review of Economic Studies*, 87(1), 240–288.

M. Gertler and K. Rogoff (1990): "North-South Lending and Endogenous Domestic Capital Market Inefficiencies," *Journal of Monetary Economics*, 26, 245–66.

S. Gilchrist, J. Sim, and E. Zakrajšek (2013): "Misallocation and Financial Market Frictions: Some Direct Evidence from the Dispersion in Borrowing Costs," *Review of Economic Dynamics*, 16(1), 159–176.

G. Gopinath, S. Kalemli-Ozcan, L. Karabarbounis, and C. Villegas-Sanchez (2017): "Capital Allocation and Productivity in South Europe," *Quarterly Journal of Economics*, 132(4), 1915–1967.

G. Gorton (1988): "Banking Panics and Business Cycles," *Oxford Economic Papers*, 40(4), 751–781.

——— (2008): "The Panic of 2007," NBER Working Papers 14358.

——— (2009): "Information, Liquidity, and the (Ongoing) Panic of 2007," *American Economic Review: Papers and Proceedings*, 99(2), 567–572.

——— (2010): "Questions and Answers about the Financial Crisis," NBER Working Papers 15787, National Bureau of Economic Research.

G. Gorton and A. Metrick (2012): "Securitized Banking and the Run on Repo," *Journal of Financial Economics*, 104(3), 425–451.

G. Gorton and G. Ordoñez (2020): "Good Booms, Bad Booms," *Journal of the European Economic Association*, 18(2), 618–665.

——— (2023): *Macroeconomics and Financial Crises*. Princeton University Press.

R. Greenwood, S. Hanson, A. Schleifer, and J. Sørensen (2022): "Predictable Financial Crises," *Journal of Finance*, 77(2), 863–921.

J. Griffin (2021): "Ten Years of Evidence: Was Fraud a Force in the Financial Crisis?," *Journal of Economic Literature*, 59(4), 1293–1321.

M. Grimm, Ò. Jordà, M. Schularick, and A. Taylor (2023): "Loose Monetary Policy and Financial Instability," NBER Working Papers 30958.

S. Gupta, A. Kangur, C. Papageorgiou, and A. Wane (2014): "Efficiency-Adjusted Public Capital and Growth," *World Development*, 57, 164–178.

F. Hayashi and E. Prescott (2002): "The 1990s in Japan: A Lost Decade," *Review of Economic Dynamics*, 5(1), 206–235.

R. Hodrick and E. Prescott (1997): "Postwar US Business Cycles: An Empirical Investigation," *Journal of Money, credit, and Banking*, pp. 1–16.

D. Ikeda (2022): "Monetary Policy, Inflation, and Rational Asset Price Bubbles," *Journal of Money, Credit and Banking*, 54(6), 1569–1603.

D. Ikeda and T. Kurozumi (2019): "Slow Post-financial Crisis Recovery and Monetary Policy," *American Economic Journal: Macroeconomics*, 11(4), 82–112.

IMF (2021): *IMF Investment and Capital Stock Dataset, 1960-2019, available online at Capital Stock Dataset*.

T. Ito and F. Mishkin (2006): "Two Decades of Japanese Monetary Policy and the Deflation Problem," in *Monetary Policy with Very Low Inflation in the Pacific Rim*, pp. 131–202. University of Chicago Press.

R. Jankowitsch, F. Nagler, and M. Subrahmanyam (2014): "The Determinants of Recovery Rates in the US Corporate Bond Market," *Journal of Financial Economics*, 114(1), 155–177.

U. Jermann and V. Quadrini (2012): "Macroeconomic Effects of Financial Shocks," *American Economic Review*, 102(1), 238–71.

G. Jiménez, E. Kuvshinov, J-P. Peydró, and B. Richter (forthcoming): "Monetary Policy, Inflation, and Crises: New Evidence from History and Administrative Data," *Journal of Finance*.

G. Jiménez, S. Ongena, J-L. Peydró, and J. Saurina (2014): "Hazardous Times for Monetary Policy: What do Twenty-three Million Bank Loans Say about the Effects of Monetary Policy on Credit Risk-taking?," *Econometrica*, 82(2), 463–505.

Ò. Jordà, K. Knoll, D. Kuvshinov, M. Schularick, and A. Taylor (2019): "The Rate of Return on Everything, 1870–2015," *The Quarterly Journal of Economics*, 134(3), 1225–1298.

Ò. Jordà, M. Schularick, and A. Taylor (2013): "When Credit Bites Back," *Journal of Money, Credit and Banking*, 45(s2), 3–28.

——— (2017): "Macrofinancial History and the New Business Cycle Facts," *NBER Macroeconomics annual*, 31(1), 213–263.

Ò. Jordà, S. Singh, and A. Taylor (2024): "The Long-Run Effects of Monetary Policy," *The Review of Economics and Statistics*, pp. 1–49.

A. Khan and J. Thomas (2013): "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity," *Journal of Political Economy*, 121(6), 1055–1107.

N. Kiyotaki and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105(2), 211–247.

L. Laeven and F. Valencia (2018): "Systemic Banking Crises Revisited," *IMF Working Papers*.

F. Longstaff, S. Mithal, and E. Neis (2005): "Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market," *The Journal of Finance*, 60(5), 2213–2253.

Guido Lorenzoni (2008): "Inefficient Credit Booms," *The Review of Economic Studies*, 75(3), 809–833.

C. Manea (2020): "Monetary Policy with Financially Constrained and Unconstrained Firms," Essays in Monetary Economics, PhD thesis, Universitat Pompeu Fabra.

G. Mankiw (1986): "The Allocation of Credit and Financial Collapse," *Quarterly Journal of Economics*, 101, 455–470.

D. Martinez-Miera and R. Repullo (2017): "Search for Yield," *Econometrica*, 85(2), 351–378.

A. Mas-Colell, M. Whinston, and J. Green (1995): *Microeconomic Theory*. Oxford University Press.

A. Mian and A. Sufi (2017): "Fraudulent Income Overstatement on Mortgage Applications during the Credit Expansion of 2002 to 2005," *Review of Financial Studies*, 30(6), 1831–1864.

V. Midrigan and D. Y. Xu (2014): "Finance and Misallocation: Evidence from Plant-Level Data," *American Economic Review*, 104(2), 422–58.

F. Mishkin (1991): "Asymmetric Information and Financial Crises: A Historical Perspective," in *Financial Markets and Financial Crises*, pp. 69–108. University of Chicago Press.

——— (1999): "Lessons from the Asian crisis," *Journal of International Money and Finance*, 18(4), 709–723.

B. Moll (2014): "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?," *American Economic Review*, 104, 3186–3221.

K. Okina, M. Shirakawa, and S. Shiratsuka (2001): "The Asset Price Bubble and Monetary Policy: Japan's Experience in the Late 1980s and the Lessons," *Monetary and Economic Studies (special edition)*, 19(2), 395–450.

P. Ottonello (forthcoming): "Capital Unemployment," *Review of Economic Studies*.

N. Oulton and M. Sebastiá-Barriel (2016): "Effects of Financial Crises on Productivity, Capital and Employment," *Review of Income and Wealth*, 63.

P. Paul (2023): "Historical Patterns of Inequality and Productivity around Financial Crises," *Journal of Money, Credit and Banking*, 55(7), 1641–1665.

M. Piazzesi and M. Schneider (2008): "Inflation Illusion, Credit, and Asset Prices," in *Asset Prices and Monetary Policy*. University of Chicago Press.

T. Piskorski, A. Seru, and J. Witkin (2015): "Asset Quality Misrepresentation by Financial Intermediaries: Evidence from the RMBS Market," *Journal of Finance*, LXX(6), 2635–2678.

C. Romer and D. Romer (2017): "New Evidence on the Aftermath of Financial Crises in Advanced Countries," *American Economic Review*, 107(10), 3072–3118.

——— (2019): "Fiscal Space and the Aftermath of Financial Crises: How it Matters and Why," *Brookings Papers on Economic Activity*.

J. Rotemberg (1982): "Monopolistic Price Adjustment and Aggregate Output," *Review of Economic Studies*, 49(4), 517–531.

M. Schularick and A. Taylor (2012): "Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises, 1870–2008," *American Economic Review*, 102(2), 1029–1061.

M. Schularick, L. Ter Steege, and F. Ward (2021): "Leaning Against the Wind and Crisis Risk," *American Economic Review: Insights*, 3(2), 199–214.

F. Smets and R. Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

J. Stein (2013): "Overheating in Credit Markets: Origins, Measurement, and Policy Responses," Speech given at the symposium on Restoring Household Financial Stability After the Great Recession, Federal Reserve Bank of St. Louis, St. Louis, Missouri.

J. Stiglitz and A. Weiss (1981): "Credit Rationing in Markets with Imperfect Information," *American Economic Review*, 71, 393–410.

J. Taylor (1993): "Discretion Versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy*, 39, 195–214.

——— (2011): "Macroeconomic Lessons from the Great Deviation," in *NBER Macroeconomics Annual*, ed. by D. Acemoglu, and M. Woodford, vol. 25, p. 387–95. University of Chicago Press.

Appendix

A Data

Table A.1: Overview of the Data Used in Figure 1

Variable	Database	Definition	AU	BE	CA	FI	FR	DE	DK	IE	IT	JP	NL	NO	PT	ES	SE	CH	GB	US
Credit	JSTa	tloans/cpi	1870	1855	1870	1870	1900	1870	1870	1932	1870	1874	1900	1870	1870	1900	1870	1870	1880	1880
Asset Price	GFD	stock price index/cpi	1870	1870	—	1912	1870	1959	1870	1870	1888	1984	1870	1914	1931	1870	1901	1899	1870	1871
Capital Stock	IMF	kpriv_n/cpi	1960	1960	1961	1960	1960	1960	1971	1960	1960	1960	1960	1962	1977	1964	1960	1965	1960	1960
TFP	JSTb	adjusted TFP	1890	1913	1890	1890	1890	1890	1890	1890	1890	1890	1890	1890	1890	1890	1890	1890	1890	1890
Output/Capital	IMF	GDP_n/kpriv_n	1960	1960	1961	1960	1960	1960	1971	1960	1960	1960	1960	1962	1977	1964	1960	1965	1960	1960
Real Equity Return	JSTa	[(1+eq_tr)/(1+dp/100)-1] ·100	1870	1870	1976	1896	1870	1870	1873	—	1870	1886	1900	1881	1871	1900	1871	1901	1871	1872
Inflation Rate	JSTa	dp ≡ (cpi-cpi(-1))/cpi(-1)·100	1870	1870	1870	1870	1870	1870	1870	1922	1870	1870	1870	1870	1870	1870	1870	1870	1870	1870
Policy Rate	JSTa	stir	1870	1870	1934	1870	1870	1870	1876	1920	1870	1879	1870	1870	1870	1870	1870	1870	1870	1870
Output	JSTa	gdp/cpi	1870	1870	1870	1870	1870	1870	1870	1922	1870	1875	1870	1870	1870	1870	1870	1870	1870	1870
Cons. Price Index	JSTa	cpi	1870	1870	1870	1870	1870	1870	1870	1922	1870	1875	1870	1870	1870	1870	1870	1870	1870	1870

Notes: The table provides an overview of the annual data used to build Figure 1. For each variable, it reports the source database, the definition of each variable based on the series in the source database, the country coverage, and the country-specific starting dates in JST. JSTa: the latest update of the Jordà et al. (2017) Macrohistory Database (R.6); GFD: Global Financial Data; IMF: IMF Investment and Capital Stock Dataset 1960–2019. JSTb: Jordà et al. (2024); Adjusted TFP: total factor productivity adjusted for labor utilization from Jordà et al. (2024); we thank Òscar Jordà, Sanjay Singh, and Alan Taylor for sharing these utilization–adjusted TFP data with us. All series end in 2020, except those from the IMF which end in 2019. AU: Austria, BE: Belgium, CA: Canada, FI: Finland, FR: France, DE: Germany, DK: Denmark, IE: Ireland, IT: Italy, JP: Japan, NL: The Netherlands, NO: Norway, PT: Portugal, ES: Spain, SE: Sweden, CH: Switzerland, GB: Great Britain, US: United States of America.

Table A.2: JST Crisis Chronology

AU	BE	CA	FI	FR	DE	DK	IE	IT	JP	NL	NO	PT	ES	SE	CH	GB	US
1893	1870	1907	1877	1882	1873	1877	**2008**	1873	1871	1921	1899	1890	1883	1878	1870	1890	1873
1989	1876		1900	1889	1891	1885		1887	1890	**2008**	1922	1920	1890	1907	1910	**1974**	1893
	1885		1921	1930	1901	1908		1893	1901		1931	1923	1913	1922	1931	**1991**	1907
	1925		1931	**2008**	1931	1921		1907	1907		**1988**	1931	1920	1931	**1991**	**2007**	1930
	1931		**1991**		**2008**	**1987**		1921	1920			**2008**	1924	**1991**	**2008**		**1984**
	1934					**2008**		1930	1927				1931	**2008**			**2007**
	1939							1935	**1997**				**1977**				
	2008							**1990**					**2008**				
								2008									

Notes: Crisis chronology (first year of each crisis) as reported in the latest update of the Jordà et al. (2017) Macrohistory Database (R.6). Database series: JSTcrisis. Crisis episodes are determined based on narratives and defined as "instances of major bank failures, banking panics, substantial losses in the banking sector, significant recapitalization, and/or significant government intervention". AU: Austria, BE: Belgium, CA: Canada, FI: Finland, FR: France, DE: Germany, DK: Denmark, IE: Ireland, IT: Italy, JP: Japan, NL: The Netherlands, NO: Norway, PT: Portugal, ES: Spain, SE: Sweden, CH: Switzerland, GB: Great Britain, US: United States of America. Bold years: sample of *post*–WW2 crises used in Figure 1.

B Summary of the Model

Our model can be summarized by the 13 equations in Table B.1, which can in turn be decomposed into two distinct blocks: the non–financial block (equations [1.]–[12.]) and the financial block (equation [13.]).

Table B.1: Equations of the Model

Non–Financial Block/Sector	
[1.]	$Z_t = \mathbb{E}_t \left\{ \Lambda_{t,t+1}(1+r_{t+1}) \right\}$
[2.]	$1 = \mathbb{E}_t \left\{ \Lambda_{t,t+1}(1+r^q_{t+1}) \right\}$
[3.]	$\frac{W_t}{P_t} = \chi N_t^{\varphi} C_t^{\sigma}$
[4.]	$Y_t = \hat{A}_t K_t^{\alpha} N_t^{1-\alpha} \quad \text{with } \hat{A}_t \equiv A_t \omega_t^{\alpha}$
[5.]	$\frac{W_t}{P_t} = \frac{\epsilon}{\epsilon-1} \frac{(1-\alpha)Y_t}{\mathscr{M}_t N_t}$
[6.]	$r^q_t + \delta = \frac{\epsilon}{\epsilon-1} \frac{\alpha Y_t}{\mathscr{M}_t K_t}$
[7.]	$(1+\pi_t)\pi_t = \mathbb{E}_t \left(\Lambda_{t,t+1} \frac{Y_{t+1}}{Y_t}(1+\pi_{t+1})\pi_{t+1} \right) - \frac{\epsilon-1}{\varrho} \left(\frac{\mathscr{M}_t - \frac{\epsilon}{\epsilon-1}}{\mathscr{M}_t} \right)$
[8.]	$1+i_t = \frac{1}{\beta}(1+\pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y}$
[9.]	$Y_t = C_t + I_t + \frac{\varrho}{2} Y_t \pi_t^2$
[10.]	$\Lambda_{t,t+1} \equiv \beta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}$
[11.]	$1 + r_t = \frac{1+i_{t-1}}{1+\pi_t}$
[12.]	$K_{t+1} = I_t + (1-\delta)K_t$
Financial Block/Sector (Credit–market Equilibrium)	
[13.]	$\omega_t = \begin{cases} 1 \text{ (Normal times)} & \text{if } r^q_t \geq \frac{(1-\theta)(1-\delta)\mu}{1-\mu} - \delta \\ 1-\mu \text{ (Crisis)} & \text{Otherwise} \end{cases}$

The financial block (equation [13.]) determines the equilibrium of the credit market and, thereby, the fraction ω_t of the economy's capital stock that is used productively, given the economy's return on equity r^q_t. Absent financial frictions, $\omega_t = 1$ regardless of r^q_t.

The non–financial block (equations [1.]–[12.]) corresponds to the standard NK model with capital accumulation, except that aggregate productivity $\hat{A}_t \equiv A_t \omega_t^{\alpha}$ consists of two components (equation [4.]): the usual exogenous component A_t and an additional component ω_t^{α}. The latter component is endogenous and captures the effect of capital reallocation (and the credit market) on aggregate productivity.

Altogether, the two blocks give rise to a feedback loop between the financial and the non–financial sectors, which eventually pins down the values of ω_t and r^q_t in the general equilibrium: given ω_t, equations [1.]–[12.] determine r^q_t; and given r^q_t, equation [13.] determines ω_t.

Online Appendix – Not For Publication

Monetary Policy and Endogenous Financial Crises

F. Boissay* F. Collard† J. Galí‡ C. Manea§

*Bank for International Settlements; email: frederic.boissay@bis.org. †Toulouse School of Economics (CNRS); email: fabrice.collard@gmail.com. ‡CREI, Universitat Pompeu Fabra and Barcelona School of Economics; email: jgali@crei.cat. §Bank for International Settlements; email: cristina.manea@bis.org.

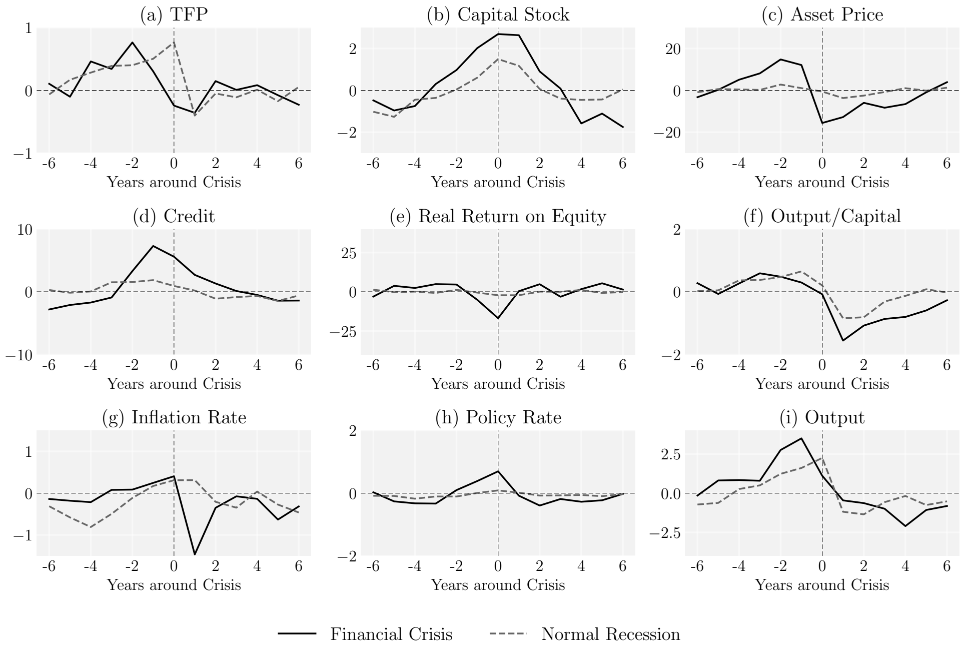
Data Availability Statement

The data and code underlying this article are available in Zenodo at **10.5281/zenodo.16947470**.

A Stylized Facts of Financial Crises: Full JST Sample

Figure A.1 shows the dynamics of the same variables as in Figure 1 over the full sample of crises in Jordà et al. (2017), *i.e.* over the period 1870–2020.

Figure A.1: Median Dynamics Around Financial Crises: 1870–2020

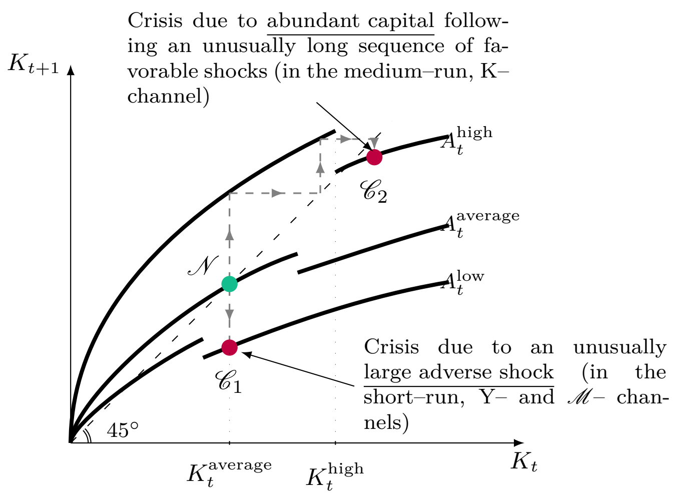


Notes: Same as Figure 1, for the full sample period (1870–2020).

B Financial Crises: Polar Types and Multiple Causes

Figure B.1 provides a stylized representation of the optimal capital accumulation decision rule, which expresses K_{t+1} as a function of the state variables K_t and A_t.

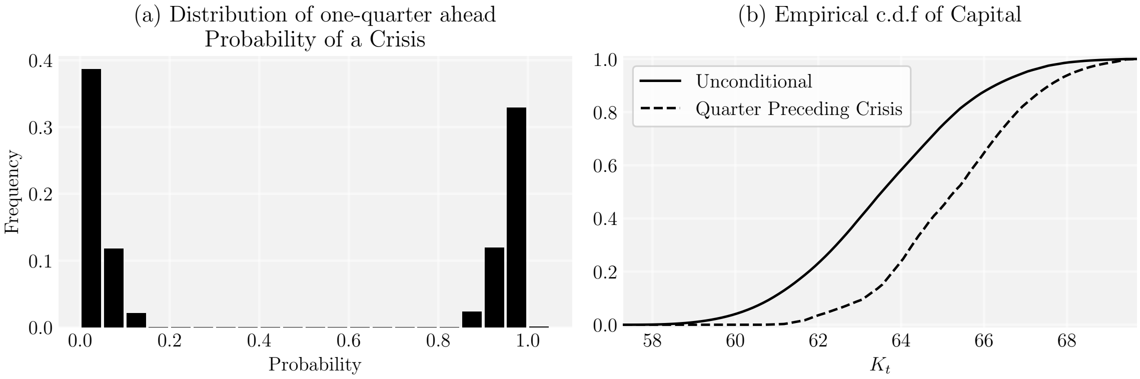
Figure B.1: Optimal Decision Rules $K_{t+1}(K_t, A_t)$ and Two Polar Types of Crisis



Notes: Stylized representation of the optimal decision rule for the capital stock.

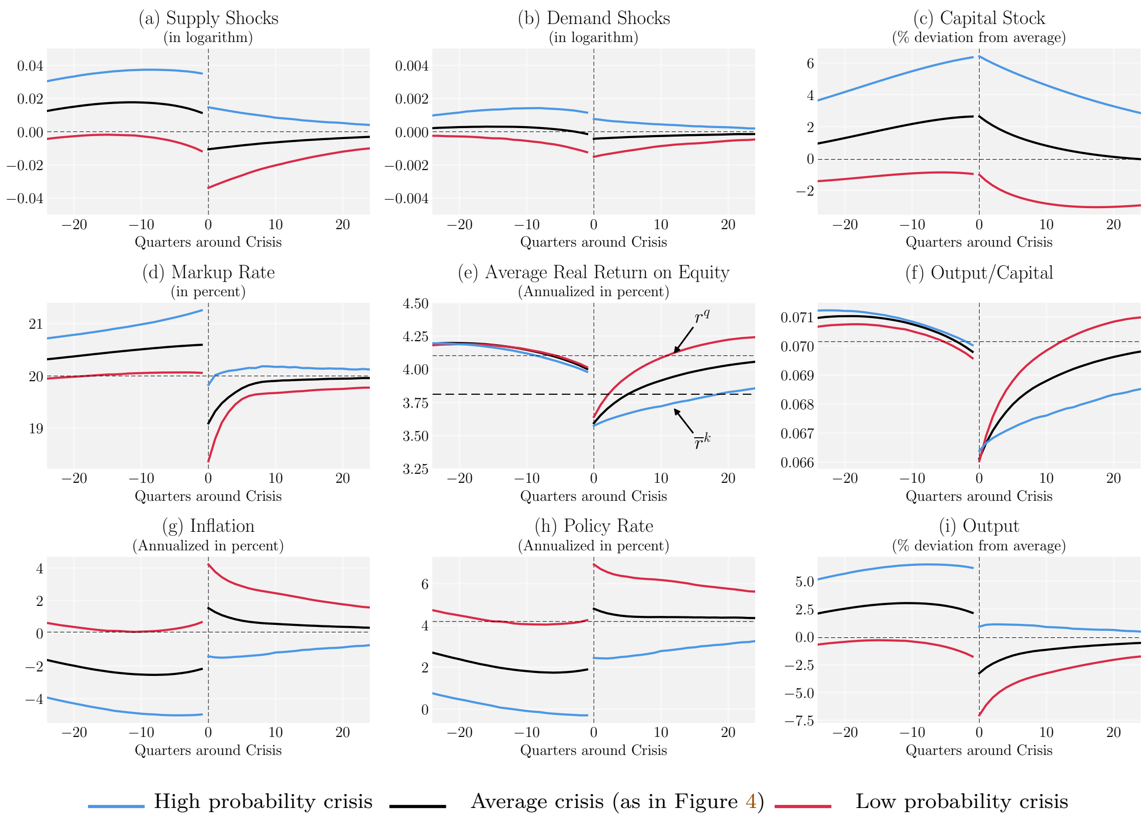
During a crisis, the household dis–invests to consume, which induces a fall in the capital stock, as captured by the discontinuous breaks in the decision rules.

Figure B.2: Crises with High *versus* Low *ex ante* Probability



Notes: Panel (a): Ergodic distribution of the one–quarter ahead probability of a crisis in the quarter that precedes crises in the TR93 economy. The *ex ante*, one–quarter ahead probability of a crisis is defined as $\mathbb{E}_{t-1}\left(\mathbb{1}\{r_t^k < \bar{r}^k\}\right)$, where $\mathbb{1}\{\cdot\}$ is equal to one when the inequality holds (*i.e.* there is a crisis) and to zero otherwise (see condition (29)). Panel (b): Ergodic cumulative distribution of the capital stock in the TR93 economy, unconditional (solid line) and conditional on being in a crisis next quarter (dashed line).

Figure B.3: Crisis Dynamics: High *versus* Low *ex ante* Probability



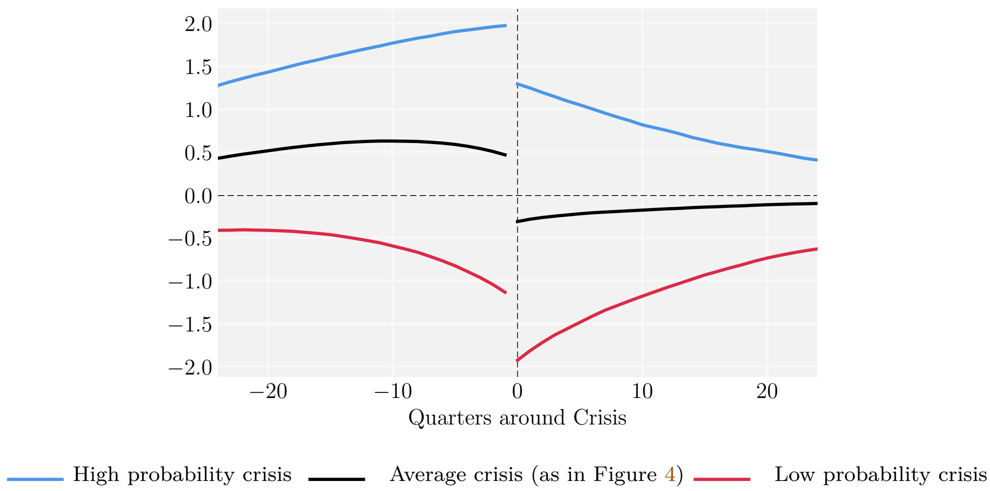
Notes: Simulations for the TR93 economy. Average dynamics of the economy around the beginning of all crises (black line, as in Figure 4) and around the beginning of crises with a low *versus* high *ex ante* crisis probability. A crisis has a low (high) *ex ante* probability if its one–quarter ahead probability in the quarter that precedes it is in the bottom (top) decile of its unconditional distribution. For the evolution of asset prices, see Figure B.4. Without loss of generality, for each variable, we report the dynamics in level or in percentage deviation from average — whichever is more appropriate.

There are two polar types of crisis. The first is triggered by a large adverse shock. For an average level of capital stock K_t^{average}, a crisis breaks out when, after the shock, the return on capital r_t^k of productive firms falls below $\bar{r}^k$ (see Proposition 2). In Figure B.1, this is the case in equilibrium $\mathscr{C}_1$, where aggregate productivity A_t falls from A_t^{average} to A_t^{low}. Figure 6 in the main text indicates that such crises tend to have a relatively low *ex ante* (one–quarter ahead) probability and, in that sense, to be "unpredictable".

The other polar type of crisis breaks out on the back of an investment boom. Following a long period of high productivity A_t^{high}, the household accumulates savings that fuel an investment boom. All else equal, the increase in capital stock reduces the return on capital r_t^k of productive firms until it falls below $\bar{r}^k$. The crisis breaks out as K_t exceeds K_t^{high}, without the economy experiencing an adverse shock, as in equilibrium $\mathscr{C}_2$. Figure 6 in the main text indicates that such a crisis tends to have a relatively high *ex ante* probability and, in that sense, to be "predictable".

In the stochastic steady state of our model, crises can be seen as different blends of the two polar types. To illustrate this heterogeneity, we report in Figure B.2 the distribution of the one–quarter ahead (*ex ante*) probability of a crisis (panel (a)) in the quarter before a crisis (quarter −1). The distribution is bimodal. Almost 40% of the crises in the stochastic steady state have an *ex ante* probability of less than 5% (extreme left bar), while more than 30% have an *ex ante* probability above 95% (extreme right bar). Panel (b) shows that the capital stock is on average higher than its unconditional average in the quarters that precede crises.

Figure B.4: Asset Price Dynamics around High *versus* Low Probability Crises



Notes: Same experiment as in Figure B.3. In percentage deviation from average.

Figures B.3 and B.4 also show how the average dynamics around crises with a high *versus* low *ex ante* probability are very different. In line with Figure B.1, we find that low *ex ante* probability crises occur when aggregate productivity and demand shocks are negative (panels (a) and (b), dotted line), as in the case of crisis $\mathscr{C}_1$ in Figure B.1, whereas high *ex ante* probability crises occur despite the exogenous component of TFP (A_t) being above average, and follow an investment boom (panel (c), dashed line), as in the case of crisis $\mathscr{C}_2$.

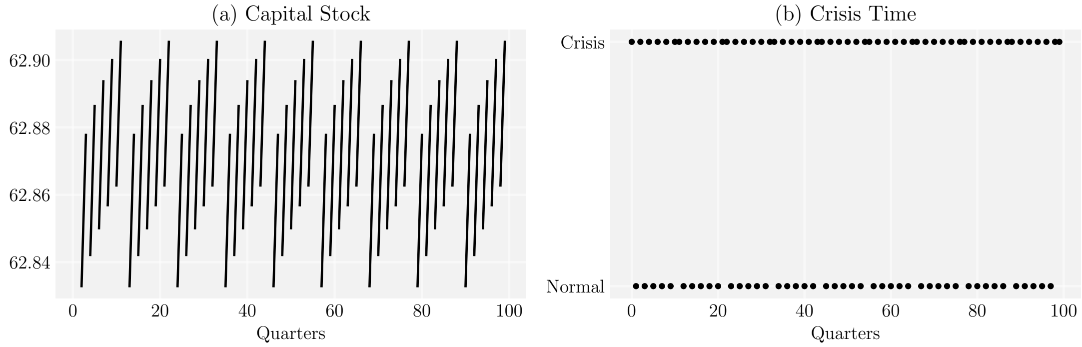
C Fully Endogenous Crises

As Figure 4 (panel (a)) suggests, financial crises in our model break out in the wake of *standard* adverse macro shocks, such as a TFP or a demand shock. These shocks are neither financial nor very large. The reason why exogenous shocks need not be very large to trigger a crisis is that our model embeds powerful internal (endogenous) dynamics, as in Boissay et al. (2016).

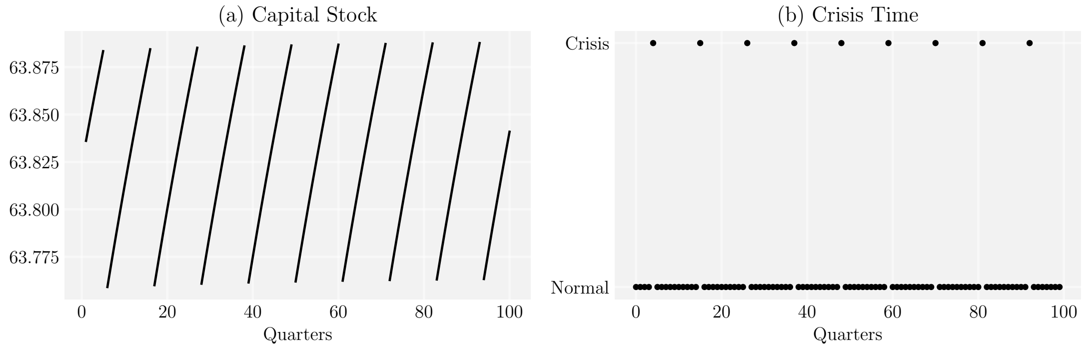
One way to illustrate these powerful internal dynamics and the endogenous aspect of financial crises is to consider our model *without* shocks. Figure C.1 shows the dynamics of the capital stock and crises in the absence of shocks when we set the parameter $\theta = 0.506$ instead of $\theta = 0.527$. In that case, our model features *deterministic* cycles and recurrent financial crises. Figure C.2 is a stylized illustration of the corresponding optimal decision rule for the capital stock.

Figure C.1: Deterministic Cycles in the Absence of Shocks

(a) Under TR93



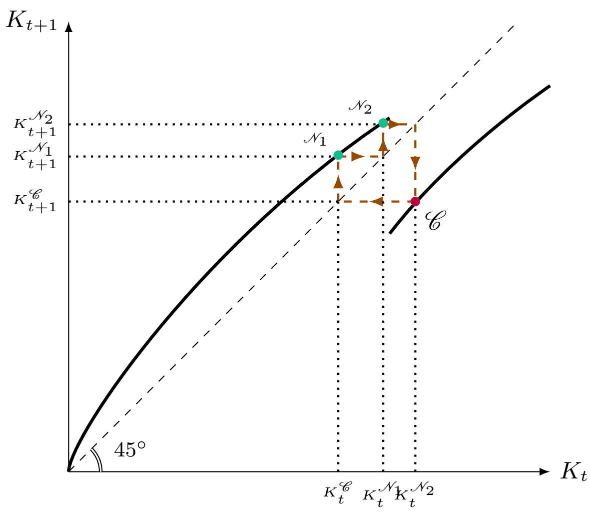
(b) Under SIT



Notes: Panels (a): deterministic dynamics of the capital stock in a version of the baseline model with $\theta = 0.506$ and without any shock. Panels (b): dummy variable that indicates when a crisis breaks out along these dynamics.

Our model's strong internal dynamics imply that exogenous shocks need not be large and are not even necessary for crises to occur in our model. Standard TFP shocks are nonetheless important to help our model match the facts and, more particularly, to account for the dynamics of productivity in the run–up to financial crises, as discussed in Section 2 and in the works of (among others) Fernald (2015) and Gorton and Ordoñez (2020).

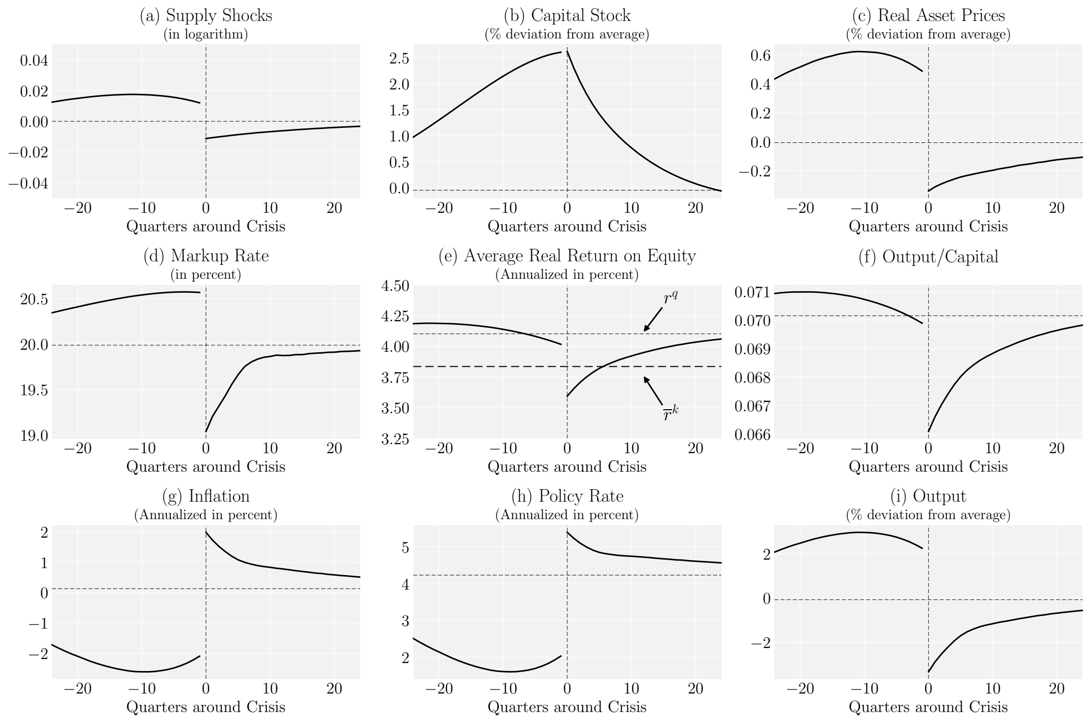
Figure C.2: Optimal Decision Rules $K_{t+1}(K_t)$
in the Absence of Shocks



Notes: Stylized illustration of the optimal decision rule in the absence of shocks.

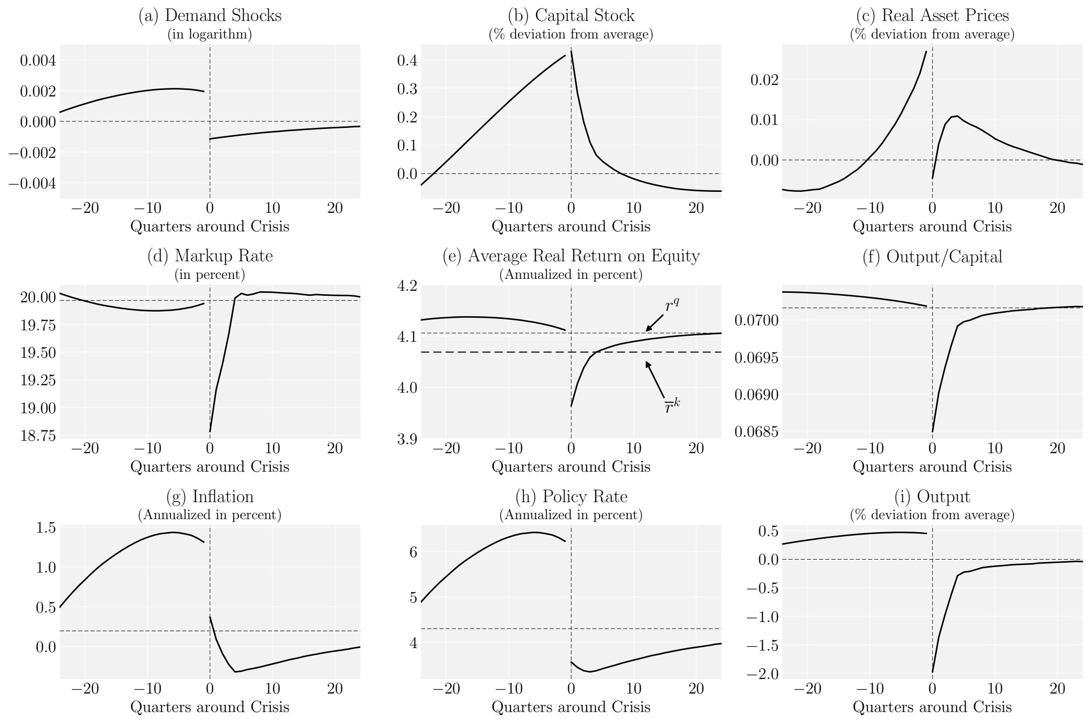
D Crisis Dynamics: Supply or Demand Shocks Only

Figure D.1: Simulated Dynamics Around Crises: Supply Shocks Only



Notes: Same as Figure 4 in an economy subject to supply shocks only. The model is re–parameterized so that the economy spends 10% of the time in crisis.

Figure D.2: Simulated Dynamics Around Crises: Demand Shocks Only



Notes: Same as Figure 4 in an economy subject to demand shocks only. The model is re–parameterized so that the economy spends 10% of the time in crisis.

E Taylor Rule with Expected Inflation

This section presents the statistics and dynamics of financial crises in a model where the central bank targets *expected* inflation instead of *current* inflation, according to the following Taylor–type rule:

$$1 + i_t = \frac{1}{\beta}(1 + \mathbb{E}_t[\pi_{t+1}])^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y}$$

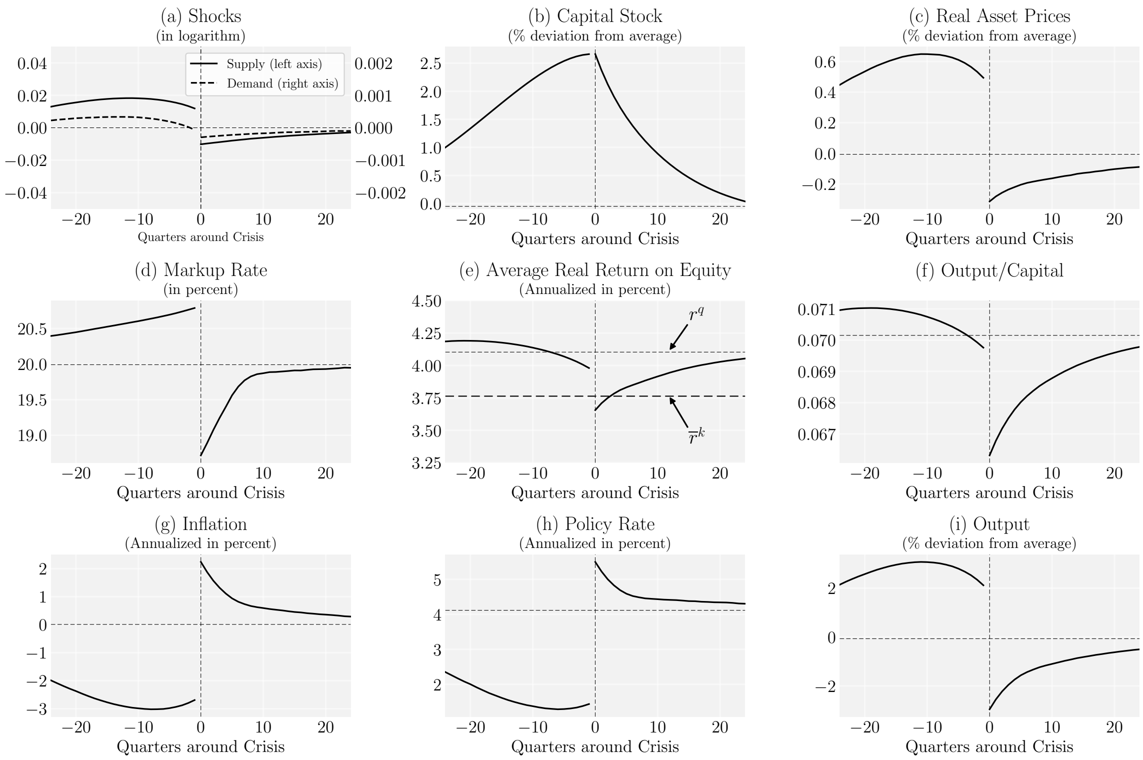
Table E.1 and Figures E.1 and E.2 show that our results are essentially the same as in our baseline model. These results suggest that our analysis is robust to considering the alternative monetary policy rule above.

Table E.1: Economic Performance and Welfare Under TR93

	Rule parameters			Model with Financial Frictions					Frictionless
	ϕ_π	ϕ_y	ϕ_r	Time in Crisis (in %)	Length (quarters)	Output Loss (in %)	Std(π_t) (in pp)	Welfare Loss (in %)	Welfare Loss (in %)
Current Inf.	1.5	0.125	–	[10]	4.7	6.0	0.9	0.58	0.33
Expected Inf.	1.5	0.125	–	[10]	5.0	5.8	1.1	0.68	0.40

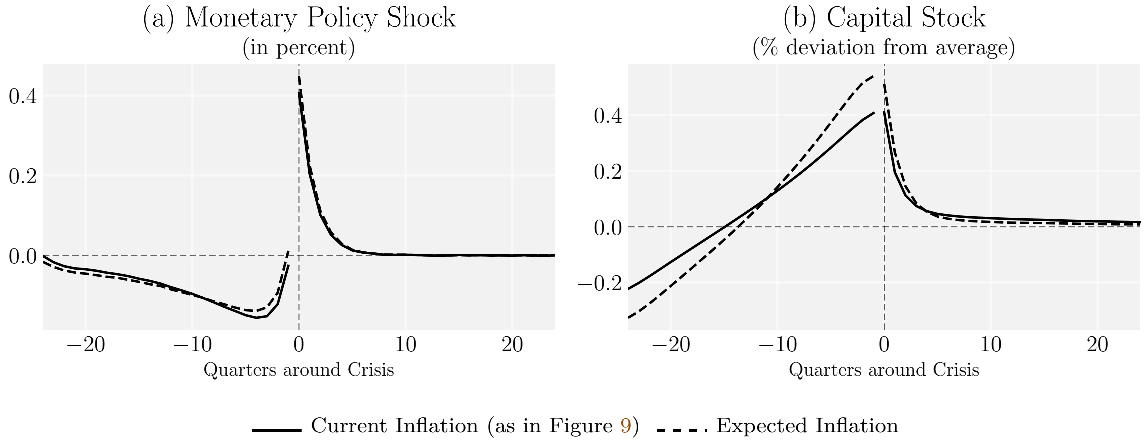
Notes: Same statistics as in Table 2. For the purpose of comparison, parameter θ of the model with expected inflation targeting was set so that the economy also spends 10% of the time in a crisis in that case ($\theta = 0.533$).

Figure E.1: Average Crisis Dynamics Under Expected Inflation Targeting



Notes: Same as Figure 4 when the Taylor rule features expected inflation. The model is re–parameterized so that the economy spends 10% of the time in crisis.

Figure E.2: Monetary Policy Shocks: Current vs Expected Inflation Targeting

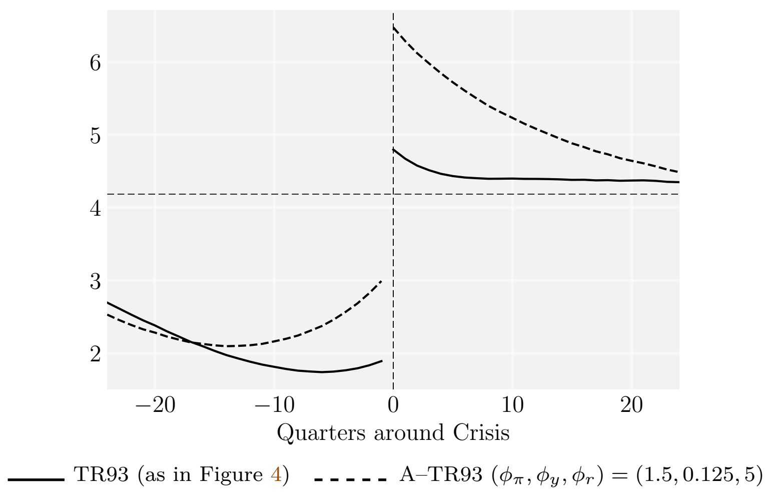


Notes: Same as Figure 9.

F Augmented Taylor Rule in a Disinflationary Boom

Figure F.1 below shows the evolution of the monetary policy rate under TR93 and A–TR93 rules during a disinflationary boom. To fix ideas, the disinflationary boom considered is the same as the one that precedes the average financial crisis in our baseline model, *i.e.* under TR93 (see Figure 4). The comparison of the two policy rate paths shows that, as expected, the central bank is more restrictive toward the end of the boom (*i.e.* between quarters −16 and 0) under A–TR93 than under TR93.

Figure F.1: Policy Rate under a Standard *versus* Augmented Taylor Rule



Notes: The model is solved under the assumption that the central bank follows either TR93 or A–TR93 (with parameters $(\phi_\pi, \phi_y, \phi_r) = (1.5, 0.125, 5)$). In the latter case, the counterfactual dynamics are derived by feeding the model with the same sequence of aggregate shocks as those that lead to a crisis under TR93 (Figure 4, panel (a)), starting with the same capital stock in quarter −24.

G Discussion on the Effects of Augmented Taylor Rules

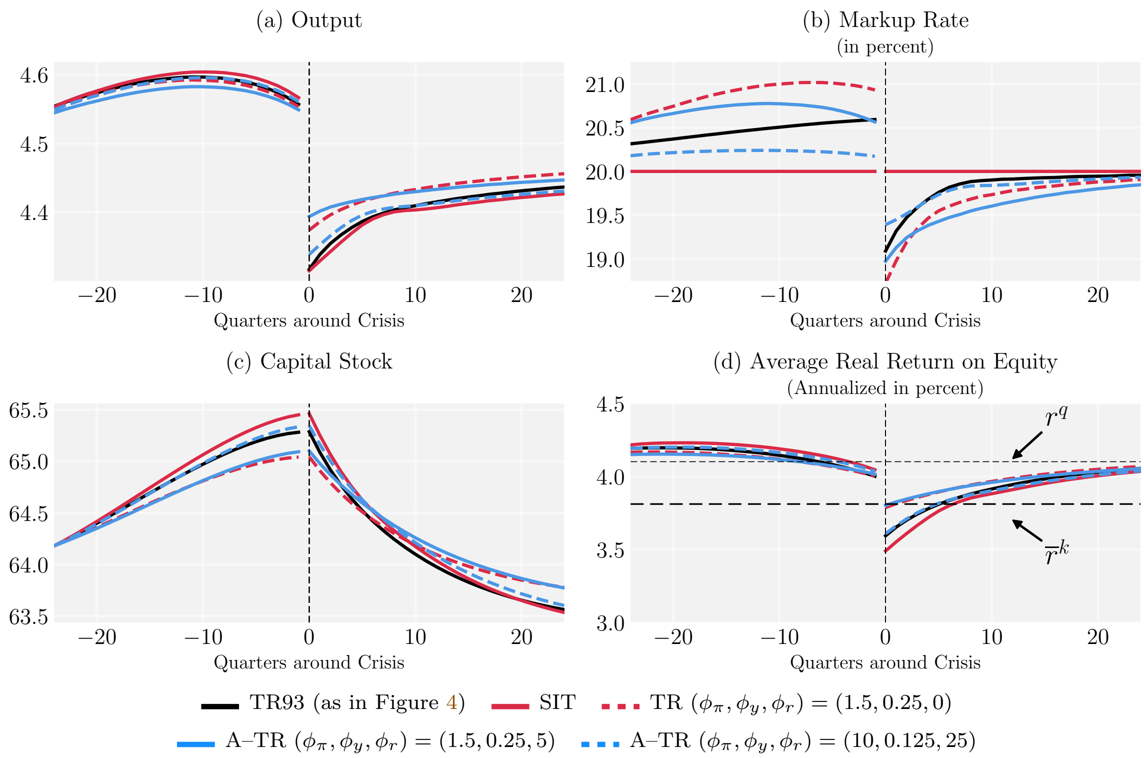
A counterfactual analysis helps gain intuition about the effects of A–TR discussed in Section 6.1. In Figure G.1 below, we compare the average dynamics of the economy under TR93 (black line) with counterfactual dynamics in economies under SIT (gray line), a Taylor–type rule with $\phi_\pi = 1.5$ and $\phi_y = 0.25$ (dashed black line), an augmented Taylor–type rule with $\phi_\pi = 1.5$, $\phi_y = 0.25$ and $\phi_r = 5$ (dashed gray line), and another one with $\phi_\pi = 10$, $\phi_y = 0.125$ and $\phi_r = 25$ as in row (9) of Table 2 (dash–dotted gray line). For the purpose of the comparison, these economies are fed with the very same sequences of shocks as those that lead to a crisis under TR93, starting with the same capital stock in quarter -24.

Consider first the dynamics of the economy from quarter -24 to quarter -1. This part of the dynamics is key to understanding how different policy responses shape the savings glut and markup externalities during the boom. Panel (d) shows that responding more aggressively to output and/or the yield gap (*i.e.* higher values of ϕ_y and/or ϕ_r; dashed red and solid blue lines) has only a modest effect on average equity returns compared to SIT or TR93. The reason is that, compared to SIT and TR93, these two policies have a larger effect on the markup but a smaller effect on capital accumulation (panels (b) and (c)). All in all, the $\mathscr{M}-$ and K–channels "offset" each other:

- Effect through the K–channel: a stronger policy response to output and/or the yield gap signals the central bank's commitment to supporting demand in a crisis and to cooling it during booms. More monetary easing in downturns reduces the need for precautionary savings, while more tightening in booms lowers investors' expected returns. Compared to TR93 or SIT, both effects work to slow capital accumulation and boost credit market resilience through the K–channel (panel (c)).

- Effect through the $\mathscr{M}$–channel: responding more aggressively to output and/or to the yield gap works to dampen inflationary pressures during booms, implying higher markups and lower credit market resilience through the $\mathscr{M}$–channel (panel (b)).

Figure G.1: Counterfactual Booms and Busts



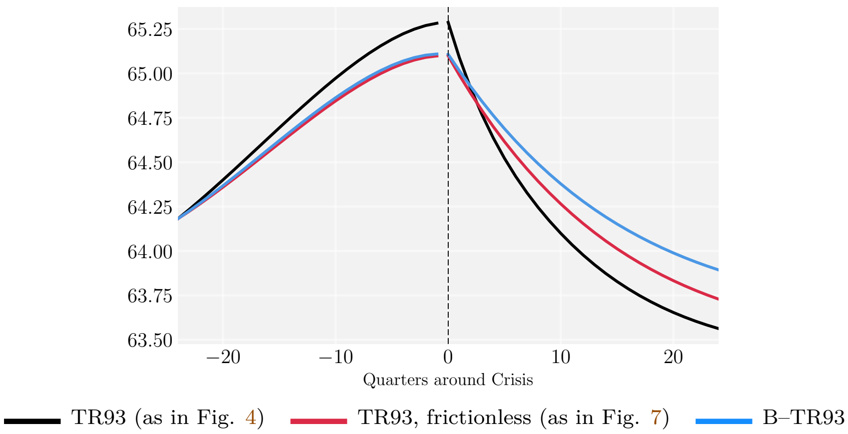
Notes: For TR93: same average dynamics as in Figure 4. For the other rules: counterfactual average dynamics, when the simulations start with the same capital stock in quarter -24 and the economy is fed with the same aggregate shocks as those that lead to a crisis under TR93 (Figure 4, panel (a)). In panel (d), the upper horizontal dashed line corresponds to the deterministic steady state value r^q of the rate of return on equity and the lower one to the crisis threshold $\bar{r}^k$ as defined in relation (29). To illustrate that the simulations start with the same capital stock, the dynamics are reported in level.

The main difference between the policy rules comes from the response of the economy at the time of the crisis, in quarter 0. While output and equity returns fall under all the rules, they fall by less when ϕ_y or ϕ_r are higher (dashed red and solid blue lines). The reason is clear: after the adverse aggregate shocks (Figure 4, panels (a) and (b)), such rules imply a bigger fall in the policy rate, which boosts aggregate demand, lifts the household's return on equity (Figure G.1, panel (d)), and helps avoid a crisis. Responding more aggressively to output and/or to the yield gap thus helps to foster financial stability by cushioning the impact of the shocks through the Y–channel.

H Capital Accumulation Under Backstop

Figure H.1 shows that capital accumulation is slower under B–TR93 than under TR93 in the run–up to financial stress episodes or financial crises. The difference reflects the household's lesser need for accumulating precautionary savings when the central bank commits to backstopping the economy which, in effect, amounts to providing households with insurance against the fall in their revenues should financial stress emerge.

Figure H.1: Capital Accumulation under B–TR93 *versus* TR93



Notes: Comparison of three economies: under TR93 with a frictional credit market (baseline, as in Figure 4); under TR93 with a frictionless credit market (as in Figure 7); and under B–TR93. For the latter two economies: counterfactual average dynamics of the capital stock, when the simulations start with the same capital stock in quarter −24 and the economy is fed with the same aggregate shocks as in the baseline. To illustrate that the simulations start with the same capital stock, the dynamics are reported in level.

I Global Solution Method

The general equilibrium is characterized by the equations reported in Table B.1. The general equilibrium solution consists of decision rules for the variables in the model. These decision rules depend on the state variables, namely the capital stock (K_t) and the exogenous shocks (denoted $s_t = (A_t, Z_t)$), and involve a threshold $\overline{K}(s_t)$ for the capital stock that determines the regime of the economy. If $K_t > \overline{K}(s_t)$ the economy is in a crisis regime. If $K_t \leq \overline{K}(s_t)$ the economy is in a normal–time regime. In turn, the regime determines the fraction of capital, ω_t, that is used productively. In normal times, $\omega_t = 1$. In a crisis, $\omega_t = 1 - \mu$. To simplify notation, we omit the time subscript t whenever it is not needed.

We solve the model numerically using a global solution method. Specifically, we discretize the state space and approximate the agents' decision rules with Chebyshev polynomials. We allow these rules to be discontinuous in order to capture regime changes between normal and crisis times.

I.1 Setup

State Space Discretization. The capital stock is assumed to take values over interval $[K_{\min}, K_{\max}]$, where $K_{\min}$ and $K_{\max}$ denote the lower and upper bounds for the capital stock. In practice, we choose these lower and upper bounds to be, respectively, 25% below and 25% above the deterministic steady–state level of capital —which lies in the normal–time regime. The distribution of the aggregate shocks is discretized using Rouwenhorst (1995)'s approach. The latter involves a Markov chain representation of the shocks, s, with $s \in \{A_1, \dots, A_{n_a}\} \times \{Z_1, \dots, Z_{n_z}\}$. We use $n_a = 5$ and $n_z = 5$ and consider n_s distinct realizations of the shocks: $s \in \{s_1, \dots, s_{n_s}\}$, with $n_s = n_a \times n_z = 25$ and transition probabilities $\varpi_{j,j'} = \mathbb{P}(s_{t+1} = s_{j'} | s_t = s_j)$.

General Equilibrium. The general equilibrium of the model can be fully solved given three "core" variables, namely: the marginal utility of consumption, denoted $mu \equiv u'(c)$; the marginal cost, denoted $mc \equiv 1/\mathscr{M}$; and the gross nominal interest rate, denoted $\hat{\imath} \equiv 1 + i$. Denoting the approximated decision rule for variable v by $G_v(K;s)$, the core decision rules are $G_{mu}(K;s)$, $G_{mc}(K;s)$, and $G_{\hat{\imath}}(K;s)$. We explain our approach to approximate these core rules in Section I.2 and the numerical algorithm in Section I.3.

For now, we derive the decision rules for the rest of the variables in the general equilibrium, given the approximated core rules $G_{mu}(K;s)$, $G_{mc}(K;s)$, and $G_{\hat{\imath}}(K;s)$, using either the variables' definition or the equations of the model in Table B.1. Using the definition of marginal utility, we obtain

$$G_C(K;s) = G_{mu}(K;s)^{-1/\sigma} \tag{I.1}$$

Equations [3.], [4.], and [5.] in Table B.1 determine the decision rule for equilibrium labor, $G_N(K;s)$. Similarly, the decision rule for aggregate output, $G_Y(K;s)$, is obtained from equation [4.]; the decision rule for the gross inflation rate, denoted by $\hat{\pi} \equiv 1+\pi$, $G_{\hat{\pi}}(K;s)$, from equation [8.]; the decision rule for the return on equity, $G_{r^q}(K;s)$, from equation [6.]; the decision rule for aggregate investment, $G_I(K;s)$, from equation [9.]; and the decision rule for next period's capital stock, $G_{K'}(K;s)$, from equation [12.]:

$$G_N(K;s) = \left(\frac{\epsilon}{\epsilon-1}\frac{1-\alpha}{\chi}G_{mc}(K;s)A(\omega K)^{\alpha}G_{mu}(K;s)\right)^{\frac{1}{\alpha+\varphi}} \tag{I.2}$$

$$G_Y(K;s) = A(\omega K)^{\alpha}G_N(K;s)^{1-\alpha} \tag{I.3}$$

$$G_{\hat{\pi}}(K;s) = \left(\frac{\beta G_{\hat{\imath}}(K;s)}{(G_Y(K;s)/Y)^{\phi_y}}\right)^{\frac{1}{\phi_\pi}} \tag{I.4}$$

$$G_{r^q}(K;s) = \frac{\epsilon}{\epsilon-1}\frac{\alpha G_{mc}(K;s)G_Y(K;s)}{K} - \delta \tag{I.5}$$

$$G_I(K;s) = G_Y(K;s) - G_C(K;s) - \frac{\varrho}{2}(G_{\hat{\pi}}(K;s)-1)^2 G_Y(K;s) \tag{I.6}$$

$$G_{K'}(K;s) = G_I(K;s) + (1-\delta)K \tag{I.7}$$

As noted above, all these calculations are performed knowing the approximated core decision rules $G_{mu}(K;s)$, $G_{mc}(K;s)$, and $G_{\hat{\imath}}(K;s)$. We now explain our approach to derive these core rules.

I.2 Decision Rule Approximation and Crisis Threshold

We approximate the decisions rules for variables $v \in \{mu, mc, \hat{\imath}\}$ using Chebyshev polynomials. To account for possible discontinuities in these decision rules, we use distinct Chebyshev polynomials for normal times (equilibrium $\mathscr{N}$) and crisis times (equilibrium $\mathscr{C}$). For $v \in \{mu, mc, \hat{\imath}\}$, we thus have

$$G_v(K;s) = \begin{cases} G_v^{\mathscr{N}}(K;s) \equiv \sum_{i=0}^{p_v} \psi_i^v(\mathscr{N},s)T_i(\nu(K)) & \text{if } K \leqslant \overline{K}(s) \text{ (normal times } \mathscr{N}) \\ G_v^{\mathscr{C}}(K;s) \equiv \sum_{i=0}^{p_v} \psi_i^v(\mathscr{C},s)T_i(\nu(K)) & \text{if } K > \overline{K}(s) \text{ (crisis times } \mathscr{C}) \end{cases} \tag{I.8}$$

In expression (I.8) above:

- $G_v^{\mathscr{N}}(K;s)$ and $G_v^{\mathscr{C}}(K;s)$ denote the optimal value of variable v when the capital stock is K and the realization of the shocks is s, in normal and crisis times, respectively;
- The crisis threshold $\overline{K}(s)$ is obtained by solving the following equation (from Corollary 2):

$$\overline{K}(s) = \frac{1-\mu}{(1-\theta)(1-\delta)\mu}\frac{\epsilon}{\epsilon-1}\alpha G_Y(\overline{K}(s),s)G_{mc}(\overline{K}(s),s) \tag{I.9}$$

  given the decision rules $G_{mc}(K,s)$ and $G_Y(K,s)$. Since the latter rules are initially unknown, we formulate an initial guess for $\overline{K}(s)$;
- $T_i(\cdot)$ denotes the Chebyshev polynomial of order i;
- $\nu(\cdot)$ is a function that maps interval $[K_{\min};\overline{K}(s)]$ in normal times, and interval $[\overline{K}(s);K_{\max}]$ in crisis times, onto $[-1;1]$. In practice, we use:

$$\nu(K) = \begin{cases} 2\frac{K-K_{\min}}{\overline{K}(s)-K_{\min}} - 1 & \text{if } K \leqslant \overline{K}(s) \text{ (normal times } \mathscr{N}) \\ 2\frac{K-\overline{K}(s)}{K_{\max}-\overline{K}(s)} - 1 & \text{if } K > \overline{K}(s) \text{ (crisis times } \mathscr{C}) \end{cases}$$

- $\psi_i^v(r,s)$ denotes the coefficient of the Chebyshev polynomial of order i used to approximate the decision rule for variable v when the economy is in regime $r \in \{\mathscr{N}, \mathscr{C}\}$ and given the realization of the shocks s;
- p_v denotes the maximal order of Chebyshev polynomials used in the approximation; in practice, we use $p_v = 9$ for $v \in \{mu, mc, \hat{\imath}\}$.

The approximated decision rules $G_{mu}(K;s)$, $G_{mc}(K;s)$, and $G_{\hat{\imath}}(K;s)$ are characterized by the set of Chebyshev coefficients

$$\Psi = \left\{\psi_i^v(r,s)\right\}_{\substack{i=0,\dots,p_v\\ v\in\{mu,mc,\hat{\imath}\}\\ r\in\{\mathscr{N},\mathscr{C}\}\\ s\in\{s_1,\dots,s_{n_s}\}}}$$

I.3 Algorithm

We now describe the algorithm used to derive the set of Chebyshev coefficients Ψ and corresponding approximated decision rules $G_{mu}(K;s)$, $G_{mc}(K;s)$, and $G_{\hat{\imath}}(K;s)$, together with the crisis threshold $\overline{K}(s)$. Throughout, we set the tolerance levels for the convergence of the Chebyshev coefficients (denoted by ε) and for the convergence of the crisis threshold (denoted by ε_k) to $\varepsilon = \varepsilon_k = 10^{-4}$. The algorithm proceeds as follows.

1. **Initialization**

   (a) We formulate an initial guess for the Chebyshev coefficients of the three core decision rules, $\psi_i^v(r,s)$, for $v = \{mu, mc, \hat{\imath}\}$, $i = 0, \dots, p_v$, $r \in \{\mathscr{N}, \mathscr{C}\}$, $s \in \{s_1, \dots, s_{n_s}\}$. In practice the initial guess for coefficient $\psi_i^v(r,s)$ is obtained by solving a log–linear approximation of the model around the deterministic steady state and projecting the log–linear decision rules on the Chebyshev polynomials $T_i(\cdot)$. The set of coefficients thus obtained characterizes the initial decision rules $G_{mu}(K;s)$, $G_{mc}(K;s)$, and $G_{\hat{\imath}}(K;s)$. Since the deterministic steady state is located in normal times, we use the same initial Chebyshev coefficients for normal and crisis times, *i.e.* $\psi_i^v(\mathscr{N},s) = \psi_i^v(\mathscr{C},s)$.

   (b) We derive the initial crisis threshold $\overline{K}(s)$ by solving equation (I.9) given the initial core decision rules $G_{mu}(K;s)$, $G_{mc}(K;s)$, and $G_{\hat{\imath}}(K;s)$ and using the decision rule $G_Y(K;s)$ as derived from (I.3).

2. **Main loop**

   (a) We construct a $2n_k$–node grid for the capital stock in state s around the crisis threshold $\overline{K}(s)$. This grid consists of n_k nodes above $\overline{K}(s)$ and n_k nodes below $\overline{K}(s)$:

   $$K_\ell(s) = \begin{cases} (\zeta_\ell + 1)\frac{\overline{K}(s) - K_{\min}}{2} + K_{\min} & \text{if } \ell = 1, \dots, n_k \text{ (Normal times)} \\ (\zeta_{\ell - n_k} + 1)\frac{K_{\max} - \overline{K}(s)}{2} + \overline{K}(s) & \text{if } \ell = n_k + 1, \dots, 2n_k \text{ (Crisis times)} \end{cases}$$

   for $\ell = 1, \dots, 2n_k$ and where

   $$\zeta_k \equiv \cos\left(\frac{(2k-1)}{2n_k}\pi\right) \text{ for } k = 1, \dots, n_k$$

   (b) Given the candidate coefficients

   $$\Psi = \left\{\psi_i^v(r,s);\ v \in \{mu, mc, \hat{\imath}\},\ r \in \{\mathscr{N}, \mathscr{C}\}, i = 0 \dots p_v,\ s \in \{s_1, \dots, s_{n_s}\}\right\}$$

   we compute the candidate approximated core decision rules $G_v(K_\ell; s_j)$ for each level of $K = K_\ell$, with $\ell = 1, \dots, 2n_k$ and each possible realization of the shock vector $s = s_j$, with $j = 1, \dots, n_s$. Given these core rules, we also derive the decisions rules for the other variables of the model, as described in (I.1)–(I.7). We obtain, in particular, next period's capital stock $G_{K'}(K_\ell; s_j)$ for $\ell = 1, \dots, 2n_k$ and $j = 1, \dots, n_s$.

(c) Given next period's capital stock $G_{K'}(K_\ell; s_j)$ and the candidate coefficients $\Psi = \{\psi_i^v(r,s);\ v \in \{mu, mc, \hat{\imath}\},\ r \in \{\mathscr{N}, \mathscr{C}\},\ i = 0 \dots p_v,\ s \in \{s_1, \dots, s_{n_s}\}\}$, we compute next period's quantities and prices that enter households' and retailers' expectations, and compute these expectations, as given in equations [1.]+[10.]+[11.], [2.]+[10.], and [7.] in Table B.1:

$$\mathscr{E}^{mu}_{\ell,j} = \beta \sum_{j'=1}^{n_s} \varpi_{j,j'} \Big[G_{mu}\left(G_{K'}(K_\ell; s_j), s_{j'}\right)\left(1 + G_{r^q}\left(G_{K'}(K_\ell; s_j), s_{j'}\right)\right) \Big] \tag{I.10}$$

$$\mathscr{E}^{\hat{\imath}}_{\ell,j} = \beta \sum_{j'=1}^{n_s} \varpi_{j,j'} \left[\frac{G_{mu}(G_{K'}(K_\ell; s_j), s_{j'})}{G_{\hat{\pi}}(G_{K'}(K_\ell; s_j), s_{j'})} \right] \tag{I.11}$$

$$\begin{aligned}\mathscr{E}^{\hat{\pi}}_{\ell,j} = \beta \sum_{j'=1}^{n_s} \varpi_{j,j'} \Big[& G_{mu}(G_{K'}(K_\ell; s_j), s_{j'}) G_Y(G_{K'}(K_\ell; s_j), s_{j'}) \times \\ & \times G_{\hat{\pi}}(G_{K'}(K_\ell; s_j), s_{j'})(G_{\hat{\pi}}(G_{K'}(K_\ell; s_j), s_{j'}) - 1) \Big]\end{aligned} \tag{I.12}$$

(d) We compute the values of the three core variables using the above expectations as well as the approximated core decision rules, $G_v(K; s)$ for $v \in \{mu, mc, \hat{\imath}\}$, and equations [1.], [2.], and [7.] in Table B.1. For each state $\{K_\ell;\ s_j\}$, for $j = 1, \dots, n_s$ and $\ell = 1, \dots, 2n_k$, we obtain the following values:

$$mu_{\ell,j} = \mathscr{E}^{mu}_{\ell,j} \tag{I.13}$$

$$mc_{\ell,j} = \frac{\epsilon - 1}{\epsilon} + \frac{\varrho}{\epsilon} \left(G_{\hat{\pi}}(K_\ell; s_j)(G_{\hat{\pi}}(K_\ell; s_j) - 1) - \frac{\mathscr{E}^{\hat{\pi}}_{\ell,j}}{G_{mu}(K_\ell; s_j) G_Y(K_\ell; s_j)} \right) \tag{I.14}$$

$$\hat{\imath}_{\ell,j} = Z_j \frac{G_{mu}(K_\ell; s_j)}{\mathscr{E}^{\hat{\imath}}_{\ell,j}} \tag{I.15}$$

(e) For each $j = 1, \dots, n_s$, we project the vectors $mu_{\{\cdot,j\}}$, $mc_{\{\cdot,j\}}$, and $\hat{\imath}_{\{\cdot,j\}}$ on the matrix of Chebyshev polynomials $\mathbf{T}(\cdot) = [T_i(\cdot)]_{i=0,\dots,p_v}$, for $v \in \{mc, mu, \hat{\imath}\}$ to obtain a new (updated) candidate vector of approximation coefficients, $\widetilde{\Psi}$. If $\|\widetilde{\Psi} - \Psi\| < \varepsilon\xi$ then a new candidate solution for the core decision rule $G_v(K, s)$ for $v \in \{mu, mc, \hat{\imath}\}$ has been found, and we go to step (2.f). Otherwise, we update the candidate solution as

$$\xi\widetilde{\Psi} + (1 - \xi)\Psi$$

where $\xi \in (0, 1]$ can be interpreted as a learning rate, and go back to step (2.b).

(f) Upon convergence of Ψ, we compute a new (updated) crisis threshold $\widetilde{\overline{K}}(s)$ that solves (I.9). If $\|\widetilde{\overline{K}}(s) - \overline{K}(s)\| < \varepsilon_k\xi_k$ then a solution for the crisis threshold $\overline{K}(s)$ has been found. Otherwise we update the threshold as

$$\xi_k\widetilde{\overline{K}}(s) + (1 - \xi_k)\overline{K}(s)$$

where $\xi_k \in (0, 1]$ can be interpreted as a learning rate on the capital threshold, and then go back to step (2.a).

I.4 Accuracy

We evaluate the accuracy of the approximated solution by the following quantities:

$$\mathscr{R}_{mu}(K_\ell; s_j) = \frac{G_C(K_\ell; s_j) - \left(\beta \sum_{j'=1}^{n_s} \varpi_{j,j'} \Big[G_{mu}\left(G_{K'}(K_\ell; s_j), s_{j'}\right)\left(1 + G_{r^q}\left(G_{K'}(K_\ell; s_j), s_{j'}\right)\right) \Big] \right)^{-\frac{1}{\sigma}}}{G_C(K_\ell; s_j)}$$

$$\mathscr{R}_{\hat{\imath}}(K_\ell; s_j) = \frac{G_C(K_\ell; s_j) - \left(\beta \frac{G_i(K_\ell; s_j)}{Z_j} \sum_{j'=1}^{n_s} \varpi_{j,j'} \left[\frac{G_{mu}(G_{K'}(K_\ell; s_j), s_{j'})}{G_{\hat{\pi}}(G_{K'}(K_\ell; s_j), s_{j'})} \right] \right)^{-1/\sigma}}{G_C(K_\ell; s_j)}$$

$$\mathscr{R}_{mc}(K_\ell;s_j) = G_{\hat{\pi}}(K_\ell;s_j)(G_{\hat{\pi}}(K_\ell;s_j)-1) + \frac{\epsilon-1}{\varrho}\left(1-\frac{\epsilon}{\epsilon-1}G_{mc}(K_\ell;s_j)\right)$$
$$-\beta\sum_{j'=1}^{n_s}\varpi_{j,j'}\left[\frac{G_{mu}(G_{K'}(K_\ell;s_j),s_{j'})}{G_{mu}(K_\ell;s_j)}\times\frac{G_Y(G_{K'}(K_\ell;s_j),s_{j'})}{G_Y(K_\ell;s_j)}\times\right.$$
$$\left.\times\, G_{\hat{\pi}}(G_{K'}(K_\ell;s_j),s_{j'})\,(G_{\hat{\pi}}(G_{K'}(K_\ell;s_j),s_{j'})-1)\right]$$

where $\mathscr{R}_{mu}(K_\ell;s_j)$ and $\mathscr{R}_{\hat{\imath}}(K_\ell;s_j)$ denote the consumption-equivalent errors that an agent would make by using the approximated expectation rather than the true rational expectation in the household Euler equation. $\mathscr{R}_{mc}(K_\ell;s_j)$ measures the error in terms of inflation. All errors are evaluated at off-grid values of the capital stock, that is, at points that were not used to compute the solution but lie within the grid span $[K_{\min},K_{\max}]$. In practice, we draw 1,000 values from a uniform distribution over $[K_{\min},K_{\max}]$. Table I.1 reports the average absolute errors,

$$E^v = \log_{10}\left(\frac{1}{n_k\times n_s}\sum|\mathscr{R}_v(K;s)|\right),$$

for $v\in\{mu,\hat{\imath},mc\}$. Intuitively, an absolute error $E^c=-5.21$ in the case $(\phi_\pi,\phi_y,\phi_r)=(1.5,0.125,0)$ means that, under TR93, the average consumption-equivalent error incurred by an agent when using the approximated decision rule—rather than the true one—amounts to approximately \$1 for every \$162,200 of consumption. The largest approximation errors in the decision rules occur at the threshold values of the capital stock, where the economy switches from normal to crisis times. The latter errors are in the order of \$1 for every \$2,500 of consumption.

Table I.1: Accuracy Measures

ϕ_π	ϕ_y	ϕ_r	E^{mu}	$E^{\hat{\imath}}$	E^{mc}
			Taylor–type Rules		
1.5	0.125	–	-5.21	-5.00	-4.82
1.5	0.250	–	-5.16	-4.78	-4.72
1.5	0.375	–	-5.06	-4.61	-4.57
2.0	0.125	–	-5.21	-4.95	-4.78
2.5	0.125	–	-5.23	-4.99	-4.77
			SIT		
$+\infty$	–	–	-5.26	–	–
			Augmented Taylor–type Rules		
1.5	0.125	5.0	-5.42	-5.25	-5.08
5.0	0.125	5.0	-5.43	-5.69	-5.15
10.0	0.125	25.0	-5.42	-5.65	-5.14
			Backstop Rules		
1.5	0.125	–	-6.07	-5.48	-5.53
$+\infty$	–	–	-5.75	–	-4.56

Notes: $E^v=\log_{10}(\frac{1}{n_k\times n_s}\sum|\mathscr{R}_v(K_\ell;s_j)|)$ is the average of the absolute difference, in terms of the level of consumption, that is obtained if agents use the approximated expectation of variable x instead of its "true" rational expectation, for $v\in\{mu,\hat{\imath},mc\}$.

J Credit Market Equilibrium Robustness: Pooling and Defaults

In our baseline model, a financial crisis takes the form of a sudden and abrupt regime switch. During a crisis, the economy switches from a regime where the credit market functions perfectly (equilibrium $\mathscr{N}$) to a regime where it completely shuts down as unproductive firms hoard their capital goods for fear that some borrowers default (equilibrium $\mathscr{C}$).

Such an abrupt switch begs the question of the existence of an intermediate regime where the economy would smoothly transition from $\mathscr{N}$ to $\mathscr{C}$. This will be the case if unproductive firms (accept to) lend capital goods even when there is a risk that they face defaults. In such an intermediate regime (if it exists), the credit market will function imperfectly but will not completely shut down.

This section presents a general version of our model in which lenders may tolerate defaults and charge a risk premium as compensation for the prospective losses. In that case, *pooling* equilibria where both productive and unproductive firms borrow may emerge. Whether lenders tolerate defaults, however, depends on the costs they incur as they "go after" delinquent borrowers in order to collect payments —henceforth, the "debt–collection/default costs".

Recall that in our baseline model a borrower may abscond with its own capital stock K_t as well as with a fraction $1-\theta$ of the borrowed capital $K_t^p - K_t$, so that the payoff from defaulting is $(1-\delta)K_t + (1-\theta)(1-\delta)(K_t^p - K_t)$ in real terms. The rest of the non–depreciated borrowed capital goods, $\theta(1-\delta)(K_t^p - K_t)$, are partly recouped by the lenders and partly dissipated in debt–collection proceedings.

Default Costs. We now explicitly model the costs of debt–collection proceedings. We assume that, to collect payments, lenders must pursue delinquent borrowers and incur a cost that amounts to a fraction

$$\kappa \geq 0$$

of the debt recovery value. A lender that lends one unit of capital to a delinquent borrower thus ultimately recoups $(1-\kappa)\theta(1-\delta)$ net of the cost κ. The parameter κ captures legal fees, attorney fees, the compensation of Chapter 7 appointed trustees, the costs of insourcing or outsourcing debt collection. In addition, we make the following assumption.

Assumption J.1. ***(Default Costs are Rebated)*** *Lenders pay the debt–collection costs to a public agent (e.g. a court) that fully rebates them to the household in the form of a subsidy on firm equity investments.*

Assumption J.1 ensures that the defaults that arise in pooling equilibria do not affect the economy through any channel other than the credit market. Since default costs are *rebated*, they only induce transfers among agents and do not alter the market clearing condition on the final goods market (*i.e.* equation [9.] in Table B.1 is unchanged). Moreover, default costs are rebated in the form of a *subsidy* proportional to equity investments (as opposed to lump–sum) which perfectly offsets the negative effects of the default costs on the household's return on equity (*i.e.* equation [2.] in Table B.1 is unchanged). Thus, under Assumption J.1, the presence of default costs does not change the equations of the non–financial block of the model (equations [1]–[12.] in Table B.1) and only affects the financial block (equation [13.]).

Preview of the Results. We show that lenders do not tolerate defaults if default costs are high enough, *i.e.* if $\kappa \geq (1-\theta)\mu/\theta(1-\mu)$. We also show that this condition is always satisfied for reasonable values for the financial parameters θ, μ, and κ. Hence, there are no pooling equilibria (and therefore no defaults) in the parameterized general version of the model. As mentioned in Footnote 17 in the main text, in the baseline version of the model we implicitly assume that the above condition on the financial parameters is satisfied, which amounts to assuming that lenders do not tolerate defaults and always impose the incentive–compatible borrowing limit (IC). In our baseline model, default costs thus only play a role *off–equilibrium.*

Roadmap. We study the possibility of defaults and pooling equilibria in four stages. In Section J.1 we characterize a pooling equilibrium in the credit market. In Section J.2, we parametrize and solve the general version of the model and show that pooling equilibria (and borrower defaults) do not emerge if one assumes plausible parameter values. In particular, *numerical* experiments suggest that such equilibria only emerge when one assumes very small default costs, *i.e.* $\kappa < 0.03$, given our parametrization targets for the time spent in crisis (10%) and the productivity loss during crises (1.8%) —recall Section 5.1. In Section J.3, we study the conditions for the existence of pooling equilibria *analytically.* We show that a necessary condition for such equilibria to exist and emerge is that $\kappa < (1-\theta)\mu/\theta(1-\mu)$ which, given our baseline parametrization of $\mu = 0.05$ and $\theta = 0.527$, implies $\kappa < 0.047$, thus confirming the numerical findings of Section J.2. In Section J.4, we further show that there is no combination of plausible values for parameters μ, θ, and κ for which the model delivers pooling equilibria. Section J.5 highlights the role of off–equilibrium defaults and the attendant prohibitive risk premium during financial crises. For the sake of completeness, the final section studies the dynamics of the economy around financial crises in a (counterfactual) version of the model with no default cost (*i.e.* with $\kappa = 0$).

J.1 Characteristics of a Pooling Equilibrium

A pooling equilibrium $\mathscr{P}$ is characterized by a loan size $K_t^p - K_t$, a probability ν_t that a given borrower repays its loan, and a credit market rate r_t^c. We consider these three characteristics in turn. To help follow the analysis, we highlight the key milestones within boxes.

J.1.1 Amount Borrowed in Equilibrium

Let λ_t denote the fraction of unproductive firms that lend in the pooling equilibrium, with

$$0 < \lambda_t < 1 \tag{J.1}$$

The value of λ_t will be determined endogenously in the general equilibrium.[J.1] Since there is a mass μ of unproductive firms and a fraction λ_t of them lend their capital stock K_t, the aggregate loan supply is equal to $\lambda_t \mu K_t$.[J.2] Given that the residual mass $(1-\lambda_t)\mu$ of unproductive firms and a mass $1-\mu$ of productive firms borrow, the pool of borrowers consists of a total mass $1-\mu+(1-\lambda_t)\mu$ of borrowers. And since lenders do not observe borrowers' types, each borrower borrows the same amount $K_t^p - K_t > 0$. The credit market clearing condition therefore reads:

$$\underbrace{\lambda_t \mu K_t}_{\text{loan supply}} = \underbrace{(1-\mu+(1-\lambda_t)\mu)(K_t^p - K_t)}_{\text{loan demand}} \Leftrightarrow \underbrace{K_t^p - K_t}_{\text{loan size}} = \frac{\lambda_t \mu}{1-\lambda_t \mu} K_t \tag{J.2}$$

which shows that the size of a loan —and therefore every borrower's leverage— increases with both the aggregate capital stock K_t and the fraction λ_t of unproductive firms that lend. Relation (J.2) can be rearranged to express the fraction ω_t of the capital stock that is used productively as a monotonically increasing function of λ_t:

$$\boxed{\omega_t \underset{(26)}{\equiv} \frac{(1-\mu)K_t^p}{K_t} = \frac{1-\mu}{1-\lambda_t \mu}} \tag{J.3}$$

The higher the mass of lenders $\lambda_t \mu$, the larger the fraction of the capital stock that is used productively.

J.1.2 Repayment Probability

Only unproductive borrowers default on their loans. Since a mass $(1-\lambda_t)\mu$ of borrowers are unproductive and default, and a mass $1-\mu$ are productive and repay their loans, the probability that a borrower repays its loan is equal to:

$$\nu_t = \frac{1-\mu}{1-\mu+(1-\lambda_t)\mu} = \frac{1-\mu}{1-\lambda_t \mu} \underset{(J.3)}{=} \omega_t \tag{J.4}$$

which shows that the repayment probability ν_t coincides with the fraction ω_t of the capital stock that is used productively in the economy. Given (J.1) and (J.4), ω_t must satisfy

$$\boxed{1-\mu < \omega_t < 1} \tag{J.5}$$

J.1.3 Credit–Market Rate

In equilibrium, the credit–market rate must satisfy the no–arbitrage conditions of both unproductive and productive firms. Consider the two types of firms in turn.

[J.1] The limit cases with $\lambda_t = 1$ and $\lambda_t = 0$ correspond to equilibria $\mathscr{N}$ (all unproductive firms lend) and $\mathscr{C}$ (none of the unproductive firms lends), respectively, and are therefore excluded from the set of possible *pooling* equilibria $\mathscr{P}$.

[J.2] Note that none of the productive firms lends in a pooling equilibrium. Indeed, in a pooling equilibrium, some unproductive firms borrow and default. Given this, a productive firm would lend only if its return on capital r_t^k was smaller than the *expected* return on loans (*i.e.* net of the losses due to defaults). But since the *expected* return on loans is necessarily *strictly smaller* than the *contractual* loan rate r_t^c when some borrowers default, this would require that $r_t^c > r_t^k$, implying that *none* of the productive firms would want to borrow, which cannot be an equilibrium.

Unproductive firms have the choice between lending their capital goods K_t or borrowing $K_t^p - K_t$ and defaulting.[J.3] If the return from lending is strictly above that from borrowing and defaulting, then none of the unproductive firms borrows and $\lambda_t = 1$; hence there is no pooling. By contrast, if the return from lending is strictly below that from borrowing, then none of the unproductive firms lends and $\lambda_t = 0$; hence there is no trade. It follows that, in a pooling equilibrium, unproductive firms must be indifferent between lending and borrowing, *i.e.* the following arbitrage condition must hold:

$$\underbrace{(1-\delta)K_t + \overbrace{(1-\theta)(1-\delta)(K_t^p - K_t)}^{\text{payoff from loan default}}}_{\substack{\text{gross return of an unproductive firm}\\ \text{that borrows (and defaults)}}} = \underbrace{\omega_t(1+r_t^c)K_t + \overbrace{(1-\omega_t)\theta(1-\delta)K_t}^{\text{recovery value}} - \overbrace{\kappa(1-\omega_t)\theta(1-\delta)K_t}^{\text{debt–collection cost}}}_{\substack{\text{gross return of an unproductive firm}\\ \text{that lends (and faces defaults)}}} \tag{J.6}$$

where the left–hand side is the payoff of an unproductive firm when it borrows, absconds, and defaults and the right–hand side is its payoff when it lends and is repaid by a fraction ω_t of its borrowers (recall (J.4)). The second term on the right–hand side is the payment that the lender recovers from delinquent borrowers. The term with κ on the right–hand side represents the debt–collection costs that the lender incurs as it collects payments from delinquent borrowers. Dividing by K_t and subtracting $1-\delta$ from both sides, one can rewrite (J.6) as

$$(1-\theta)(1-\delta)\frac{K_t^p - K_t}{K_t} = \omega_t(r_t^c + \delta) - (1-\omega_t)(1-\delta)(1-\theta) - (1-\omega_t)(1-\delta)\theta\kappa$$

and using the definition of ω_t in (26) one further obtains

$$(1-\theta)(1-\delta)\left(\frac{\omega_t}{1-\mu} - 1\right) = \omega_t(r_t^c + \delta) - (1-\omega_t)(1-\theta)(1-\delta) - (1-\omega_t)(1-\delta)\theta\kappa$$

$$\Leftrightarrow \boxed{\omega_t(r_t^c + \delta) = (1-\delta)\theta\left[\frac{(1-\theta)\mu}{\theta(1-\mu)} - \kappa\right]\omega_t + (1-\delta)\theta\kappa} \tag{J.7}$$

When $\kappa = 0$ (*i.e.* there are no debt–collection costs), the terms in κ vanish from (J.7) and the terms in ω_t drop out on both sides. Consequently, the loan rate r_t^c at which an unproductive firm is indifferent between lending *versus* borrowing is independent of the repayment probability ω_t. This observation reflects two opposite and offsetting effects of ω_t on unproductive firms' incentives in this particular case:

- On the one hand, a higher repayment probability ω_t (higher λ_t) raises the payoff from lending, which increases unproductive firms' incentive to *lend* (right–hand side of (J.6));
- On the other hand, a higher ω_t (higher λ_t) is associated with a higher aggregate loan supply and therefore a higher leverage $(K_t^p - K_t)/K_t$ in equilibrium, which increases unproductive firms' incentive to *borrow and default* (left–hand side of (J.6)).

Thus, when $\kappa = 0$, unproductive firms' payoffs from lending and borrowing move in sync, and a change in ω_t does not have any effect on the equilibrium (no–arbitrage) loan rate r_t^c.

When $\kappa > 0$, by contrast, an increase in the probability of debt repayment ω_t not only increases the expected return on the loan but also reduces the debt–collection costs, which, all else equal, tilts the balance toward lending. In that case, the equilibrium loan rate r_t^c that keeps unproductive firms indifferent between lending and borrowing must go down as ω_t goes up: r_t^c monotonically decreases with ω_t. Moreover, the higher κ, the "faster" r_t^c must decrease with ω_t for unproductive firms to remain indifferent.

A similar no–arbitrage condition holds for productive firms. From the optimization program in (19), one can see that productive firms make strictly negative profits with their borrowed capital when $r_t^k < r_t^c$, and therefore do not borrow in that case. They also make strictly positive profits when $r_t^k > r_t^c$ and therefore demand an infinite loan amount in that case. As a result, a pooling equilibrium with trade can only be achieved if productive firms break even with their loans, *i.e.* if:

$$\boxed{r_t^c = r_t^k} \tag{J.8}$$

where r_t^k is the productive firms' return on capital that prevails in the general equilibrium with pooling (if it exists).

[J.3] Recall that an unproductive firm only borrows with the intention to abscond and default. Since the payoff from keeping capital K_t idle is lower than that from borrowing and defaulting, "doing nothing" is not a relevant alternative to lending (the payoff on the left–hand side of (J.6) is higher than $(1-\delta)K_t$).

Default Risk Premium. The loan rate r_t^c that satisfies the no–arbitrage condition (J.8) can be interpreted as the *maximum* loan rate that productive firms are willing to pay. In contrast, the loan rate r_t^c that satisfies the no–arbitrage condition (J.7) can be interpreted as the *minimum* loan rate that unproductive firms require in order to lend. This loan rate compensates lenders for default risk and embeds a risk premium which increases with the default probability, $1-\omega_t$, the loss given default, $1+r_t^c-\theta(1-\delta)$, and the default cost, $\kappa\theta(1-\delta)$.

J.1.4 Household's Return on Equity in a Pooling Equilibrium

In a pooling equilibrium, the dividends that the household receives from the firms amount to (in real terms):

$$
\begin{aligned}
(1+r_t^q)K_t \equiv & (1-\lambda_t)\mu \underbrace{\left[(1-\delta)K_t+(1-\theta)(1-\delta)(K_t^p-K_t)\right]}_{\substack{\text{dividends from the unproductive}\\ \text{firms that borrow and default}}} \\
& + \underbrace{\lambda_t\mu\left[\omega_t(1+r_t^c)K_t+(1-\omega_t)\theta(1-\delta)K_t-(1-\omega_t)\theta\kappa(1-\delta)K_t\right]}_{\substack{\text{dividends from the unproductive firms that lend}\\ \text{net of debt–collection costs}}} \\
& + (1-\mu)\underbrace{\left[(1+r_t^k)K_t^p-(1+r_t^c)(K_t^p-K_t)\right]}_{\substack{\text{dividends from}\\ \text{productive firms/borrowers}}} + \underbrace{\lambda_t\mu(1-\omega_t)\theta\kappa(1-\delta)K_t}_{\substack{\text{subsidy that corrects distortions}\\ \text{due to default costs}}}
\end{aligned}
\tag{J.9}
$$

where the last term is the subsidy that the household receives from the government as compensation for the debt–collection costs (Assumption J.1). This subsidy is proportional to (real) equity investment and corrects for the negative effect of default costs on the household's return on equity and attendant distortions: in (J.9), the terms in κ cancel out. Replacing $K_t^p-K_t$ and λ_t by their respective expressions in terms of ω_t in (J.3), one can also see that all the resource transfers across firms, *i.e.* the loan payments (terms in r_t^c) and loan losses (terms in θ), cancel out. One eventually obtains:

$$
r_t^q+\delta=\omega_t(r_t^k+\delta)=\underbrace{(1-\omega_t)\cdot(-\delta)+\omega_t r_t^k}_{r_t^q}+\delta \tag{J.10}
$$

where $\omega_t\in(1-\mu,1)$. The relation (J.10) is the same as the relation (34) in the baseline model (except that $\omega_t\in(1-\mu,1)$ instead of $\omega_t\in\{1-\mu,1\}$ in a pooling equilibrium). Thus, under Assumption J.1, default costs do not affect the household's return on equity directly (*i.e.* through their costs) and, as in the baseline version of the model, r_t^q coincides with the *aggregate* return on capital. Indeed, recall that the latter is defined as the weighted average of the returns on idle capital goods ($-\delta$) and productive capital goods (r_t^k) using as weights the fraction $1-\omega_t$ of the capital stock that is kept idle and the fraction ω_t that is used productively —as emphasized by the last equality in (J.10).

J.1.5 Condition of an Equilibrium With Pooling in the Credit Market

Combining firms' no–arbitrage conditions (J.7) and (J.8), one obtains:

$$
\underbrace{\omega_t\left(r_t^k+\delta\right)}_{\substack{\text{from no–arbitrage}\\ \text{condition (J.8)}}} = \underbrace{(1-\delta)\theta\left[\frac{(1-\theta)\mu}{\theta(1-\mu)}-\kappa\right]\omega_t+(1-\delta)\theta\kappa}_{\substack{\text{from no–arbitrage}\\ \text{condition (J.7)}}}
$$

where $r_t^k=r^k(\omega_t\mid A_t,Z_t,K_t)$ is the productive firms' return on capital that prevails in the general equilibrium and is an implicit function of ω_t given the state of Nature $\{A_t,Z_t,K_t\}$ —recall expression (28). Using the relation $r_t^q+\delta=\omega_t(r_t^k+\delta)$ (see (34) or (J.10) above), one can further rewrite the above equilibrium condition as

$$
\underbrace{r_t^q+\delta}_{\substack{r^q(\omega_t|A_t,Z_t,K_t)+\delta\\ \text{from (J.8)}}} = \underbrace{(1-\delta)\theta\left[\frac{(1-\theta)\mu}{\theta(1-\mu)}-\kappa\right]\omega_t+(1-\delta)\theta\kappa}_{\substack{\varphi(\omega_t)\\ \text{from (J.7)}}} \tag{J.11}
$$

$$
\Leftrightarrow \omega_t=\frac{r_t^q+\delta-(1-\delta)\theta\kappa}{(1-\delta)\theta\left[\frac{(1-\theta)\mu}{\theta(1-\mu)}-\kappa\right]}
$$

where $r_t^q = r^q(\omega_t \mid A_t, Z_t, K_t)$ is the household's return on equity that prevails in the general equilibrium and is an implicit function of ω_t given the state of Nature $\{A_t, Z_t, K_t\}$. In any pooling equilibrium $\{\omega_t, K_t^p - K_t, r_t^c\}$, the equilibrium value of ω_t is the solution to equation (J.11) that also satisfies condition (J.5). Given the equilibrium value of $\omega_t \in (1-\mu, 1)$ (if it exists), one can then derive $K_t^p - K_t$ using the definition of ω_t in (26) and r_t^c from (J.7).

J.2 General Model With Pooling and Defaults

In the general equilibrium, the market clearing conditions (30), (31) and (32) still hold and aggregate output is still given by (33), with $\omega_t \in [1-\mu, 1]$. Pooling equilibria emerge when conditions (J.5) and (J.11) are satisfied. Even though the credit market helps to reallocate capital from unproductive to productive firms ($\omega_t > 1-\mu$), the reallocation is partial ($\omega_t < 1$) in a pooling equilibrium. In what follows, we refer to pooling equilibria as "credit–market disruptions".

The complete list of equations for the general version of the model is reported in Table J.1. This version of the model accounts for the possibility of both credit–market disruptions and financial crises (equation [13.]). The non–financial block (Equations [1.]–[12.]) is the same as that in the baseline version of the model (see Table B.1). It also corresponds to the standard NK model with capital accumulation, except that total factor productivity $\hat{A}_t$ has two components A_t and ω_t. The share of productive capital ω_t only appears in equation [4.] and affects the general equilibrium outcome through total factor productivity $\hat{A}_t$ only. As in the baseline version of the model, equations [1.]–[13.] give rise to a feedback loop between the financial– and the non–financial sectors, which pins down the value of ω_t in the general equilibrium. Given ω_t, equations [1.]–[12.] determine r_t^q; and given r_t^q, equation [13.] determines ω_t.

Solution. The numerical solution of the dynamic general equilibrium rules out coordination failures by selecting the most efficient equilibrium whenever it exists (recall Assumption 1). Accordingly, the solution is sequential:

1. We first assume that the normal–times equilibrium exists (*i.e.* that $\omega_t = 1$), derive the household's average rate of return on equity r_t^q in the general equilibrium under this assumption, and check *a posteriori* that $r_t^q + \delta \geq (1-\delta)(1-\theta)\mu/(1-\mu)$. If this condition is satisfied, then the normal–times equilibrium exists and we select it.

2. If the normal–times equilibrium does not exist, then we assume that a pooling equilibrium exists (*i.e.* that a fraction $\omega_t \in (1-\mu, 1)$ of the capital stock is used productively), derive the average rate of return on equity r_t^q under this assumption, and check *a posteriori* that $r_t^q + \delta \in ((1-\delta)(1-\theta(1-\kappa))\mu; (1-\delta)(1-\theta)\mu/(1-\mu))$. If this condition is satisfied, then the pooling equilibrium exists and we select it. In that case, the credit market features disruptions.

3. If neither the normal–times equilibrium nor the pooling equilibrium exists, the only equilibrium that remains is the crisis time equilibrium. We then conclude that there is a complete shutdown of the credit market (*i.e.* a crisis).

Parametrization. The general version of the model features one additional parameter compared to the baseline version: the default cost parameter κ. The latter, which enters equation [13.], governs the size of credit–market disruptions as well as the time that the economy spends with credit–market disruptions *versus* shutdowns. Importantly, κ does not directly affect the condition of existence of normal times: as in the baseline model, the economy is in normal times whenever $r_t^q + \delta \geq (1-\theta)(1-\delta)\mu/(1-\mu)$, a condition that only depends *directly* on parameters θ and μ. The parameter κ thus only affects the incidence of credit–market disruption and crises *indirectly* through general equilibrium feedback effects on r_t^q.

To set the value of κ, we rely on the recent work by Antill (2024), who documents that when a US borrower files for bankruptcy under Chapter 7 the appointed trustee charges creditors substantial fees to recover their debts.[J.4] As Antill (2024) reports a median legal fee of 11–13% of the debt recovery value depending on the sample considered (Table 2, p. 3604), we set $\kappa = 0.12$. To the extent that lenders incur other debt–collection costs on top of legal fees (*e.g.* the cost of hiring private debt collectors, the opportunity cost of being paid late), such fees should be regarded as a lower bound for lenders' total

[J.4] In the United States, the compensation of trustees is regulated under Chapter 7 (see legal fees).

default costs. We also set $\mu = 0.05$ to obtain a 1.8% fall in aggregate productivity due to financial frictions during a crisis (*e.g.* Gilchrist et al. (2013)). Given parameters $\kappa = 0.12$ and $\mu = 0.05$, we then set θ jointly with the standard deviations of the demand and technology shocks to match in the simulated stochastic steady state of the model the observed incidence of financial crises of 10% (Romer and Romer (2017)) and the volatility of the Hodrick–Prescott quarterly cyclical components of core inflation and output in the post–WW2 period. We obtain $\theta = 0.527$, $\sigma_a = 0.007$, and $\sigma_z = 0.001$, as in our baseline model. For these parameter values, the model does not exhibit any pooling equilibria, market disruptions or borrower defaults in any state of Nature.

Table J.1: General Version of the Model

Non–Financial Block/Sector

[1.] $Z_t = \mathbb{E}_t\left\{\Lambda_{t,t+1}(1+r_{t+1})\right\}$

[2.] $1 = \mathbb{E}_t\left\{\Lambda_{t,t+1}(1+r^q_{t+1})\right\}$

[3.] $\frac{W_t}{P_t} = \chi N_t^{\varphi} C_t^{\sigma}$

[4.] $Y_t = \hat{A}_t K_t^{\alpha} N_t^{1-\alpha}$ with $\hat{A}_t \equiv A_t \omega_t^{\alpha}$

[5.] $\frac{W_t}{P_t} = \frac{\epsilon}{\epsilon-1}\frac{(1-\alpha)Y_t}{\mathscr{M}_t N_t}$

[6.] $r^q_t + \delta = \frac{\epsilon}{\epsilon-1}\frac{\alpha Y_t}{\mathscr{M}_t K_t}$

[7.] $(1+\pi_t)\pi_t = \mathbb{E}_t\left(\Lambda_{t,t+1}\frac{Y_{t+1}}{Y_t}(1+\pi_{t+1})\pi_{t+1}\right) - \frac{\epsilon-1}{\varrho}\left(\frac{\mathscr{M}_t - \frac{\epsilon}{\epsilon-1}}{\mathscr{M}_t}\right)$

[8.] $1+i_t = \frac{1}{\beta}(1+\pi_t)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_y}$

[9.] $Y_t = C_t + I_t + \frac{\varrho}{2}Y_t\pi_t^2$

[10.] $\Lambda_{t,t+1} \equiv \beta\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}$

[11.] $1+r_t = \frac{1+i_{t-1}}{1+\pi_t}$

[12.] $K_{t+1} = I_t + (1-\delta)K_t$

Financial Block/Sector (Credit–Market Equilibrium)

[13.] $$\omega_t = \begin{cases} 1 \quad \textit{(Normal times)} & \text{if } r^q_t \geq \frac{(1-\theta)(1-\delta)\mu}{1-\mu} - \delta \\ \frac{r^q_t+\delta-(1-\delta)\theta\kappa}{(1-\delta)\theta\left[\frac{(1-\theta)\mu}{\theta(1-\mu)}-\kappa\right]} \quad \textit{(Credit–Market Disruption/Pooling Equil.)} & \text{if } \kappa < \frac{(1-\theta)\mu}{\theta(1-\mu)} \text{ and } \frac{r^q_t+\delta-(1-\delta)\theta\kappa}{(1-\delta)\theta\left[\frac{(1-\theta)\mu}{\theta(1-\mu)}-\kappa\right]} \in (1-\mu, 1) \\ 1-\mu \quad \textit{(Crisis)} & \text{Otherwise} \end{cases}$$

We conclude that for plausible values of financial parameters μ, κ, and θ, the general version of the model boils down to the baseline version considered in the main text, where lenders do not tolerate any default. In that case, financial crises take the form of a *sudden and abrupt* regime switch from equilibria $\mathscr{N}$ to $\mathscr{C}$. There is no smooth transition (*i.e.* credit–market disruptions) between $\mathscr{N}$ and $\mathscr{C}$.

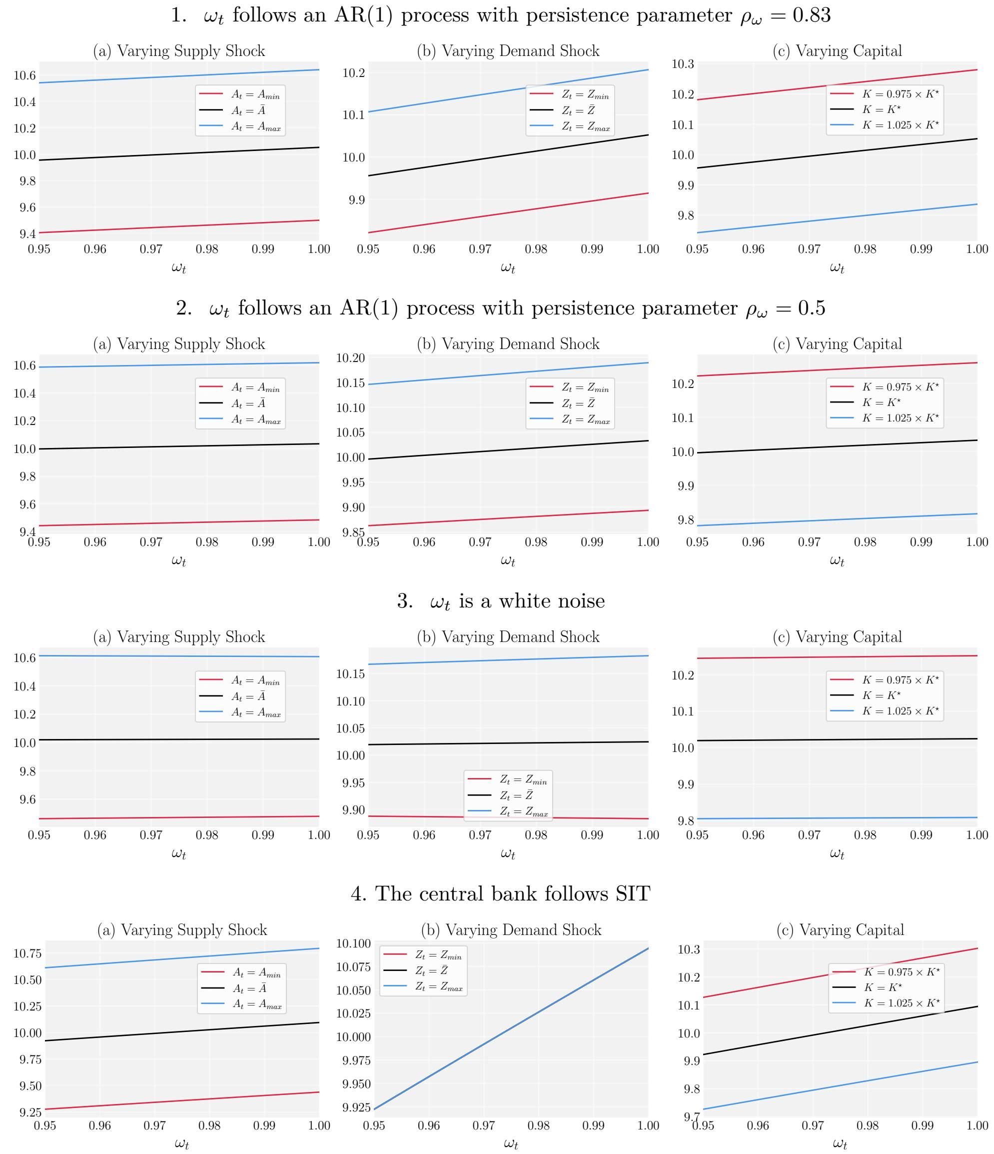
One possible explanation for the absence of pooling equilibria when $\kappa = 0.12$ is that the default costs may be "too high", making it unacceptable for lenders to tolerate defaults in any state of Nature.

To investigate this point, we consider smaller values of κ and solve the model numerically as we lower κ incrementally from 0.12 to 0 *all else remaining the same.* In particular, we keep the financial parameters $\mu = 0.05$ and $\theta = 0.527$ unchanged. We find that pooling equilibria (with credit–market disruptions and borrower defaults) only emerge for values of κ *below* 0.03. We conclude that pooling equilibria may only emerge for unrealistically small default costs, *i.e.* for default costs that are less than one fourth of their observed values. We explain these numerical results and explore the conditions for the existence of pooling equilibria formally in the next section.

J.3 Condition of Existence of an Equilibrium With Pooling and Defaults

The terms $r^q(\omega_t \mid A_t, Z_t, K_t)$ and $\varphi(\omega_t)$ on the left– and right–hand sides of (J.11) both depend on ω_t. Consider the variation of each of these terms with respect to ω_t in turn.

Figure J.1: Representation of $r^q(\omega_t \mid A_t, Z_t, K_t) + \delta$



Notes: Representations of the function $r^q(\omega_t \mid A_t, Z_t, K_t) + \delta$ based on the optimal decision rule $r^q(A_t, Z_t, K_t, \omega_t)$ obtained by solving the non–financial block of the baseline model (equations [1.]—[12.] in Table J.1), using the parameters in Table 1 and assuming that ω_t follows an exogenous (truncated) AR(1) process over the interval $[1-\mu, 1]$. Row 1.: ω_t is assumed to follow an AR(1) that emulates the endogenous process of ω_t in the stochastic steady state of the full model (equations [1.]—[13.]) for $\kappa = 0$: $\ln(\omega_t) = \rho_\omega \ln(\omega_{t-1}) + \varepsilon_t^\omega$ with $\rho_\omega = 0.83$. Rows 2. and 3.: same as in row 1., assuming $\rho_\omega = 0.5$ and $\rho_\omega = 0$. Row 4.: same as in row 1., assuming that the central bank follows SIT instead of TR93 as the policy rule. Representations for varied states of Nature: $A_t \in \{A_{min}, 1, A_{max}\}$ (panel (a)); $Z_t \in \{Z_{min}, 1, Z_{max}\}$ (panel (b)); $K_t \in \{0.975K^\star, K^\star, 1.025K^\star\}$ (panel (c)). Annualized, in percent. Similar results are obtained for an autocorrelation coefficient $\rho_\omega = 0.95$ for the data generating process of ω_t.

J.3.1 Analysis of Function $r^q(\omega_t \mid A_t, Z_t, K_t)$

The function $r_t^q = r^q(\omega_t \mid A_t, Z_t, K_t)$ is an outcome of the dynamic general equilibrium and cannot be derived analytically. To get a sense of how this function varies with ω_t, one way is to consider a version of the model that abstracts from the financial block (equation [13.] in Table J.1), to treat ω_t as an exogenous state variable, and to solve the non–financial block (equations [1.]–[12.]) of the general model assuming a data generating process for $\omega_t \in [1-\mu, 1]$. Note that the equations of the non–financial block of the general model are exactly the same as those of our baseline model, and also coincide with the equations of the standard NK model with capital accumulation.

To alleviate any concern that the exogenous process assumed for ω_t may not correspond to the "true" endogenous data generating process of ω_t in the general model (equation [1.]–[13.]), we emulate the latter by assuming that $\omega_t \in [1-\mu, 1]$ follows a (truncated) AR(1) process with the same persistence (autocorrelation coefficient of 0.83) as in the stochastic steady state of the general model with $\kappa = 0$. As a robustness check, we consider two alternative (less persistent) exogenous data generating processes for ω_t: one where ω_t follows an AR(1) process with a persistence coefficient of 0.5, and another where it is an independently and identically distributed random variable (*i.e.* is a white–noise process).

The optimal decision rules for the endogenous variables (including that for r_t^q) are functions of the state variables $\{A_t, Z_t, K_t, \omega_t\}$; accordingly, the numerical solution yields $r_t^q = r^q(A_t, Z_t, K_t, \omega_t)$. Given the state variables $\{A_t, Z_t, K_t\}$, r_t^q can also be expressed as a function of ω_t: $r_t^q = r^q(\omega_t \mid A_t, Z_t, K_t)$.

Figure J.1 (row 1.) shows $r^q(\omega_t \mid A_t, Z_t, K_t)$ for selected states of Nature $\{A_t, Z_t, K_t\}$ in the case where the process of ω_t is characterized by an autocorrelation coefficient equal to 0.83. We find that $r^q(\omega_t \mid A_t, Z_t, K_t)$ monotonically increases with ω_t, suggesting that the household's return on equity (or the aggregate return on capital) increases when a larger fraction of the capital stock is used productively. The monotonic relationship between r_t^q and ω_t does not change across states of Nature. The results for the other two processes for ω_t, reported in rows 2. and 3., are essentially the same as in row 1., *i.e.* $r^q(\omega_t \mid A_t, Z_t, K_t)$ monotonically increases with ω_t.[J.5] Finally, we also consider the case where the central bank follows a SIT rule instead of a TR93 rule and find that r_t^q still monotonically increases with ω_t (row 4.). The results remain the same throughout.

We conclude that, in the dynamic general equilibrium, r_t^q increases monotonically with ω_t regardless of the state of Nature considered. Importantly, this result applies to the non–financial block of our model, which coincides with the standard NK model with capital accumulation. It can therefore also be seen as a property of the standard NK model with capital accumulation and is not specific to our model. This property of the standard NK model basically means that the aggregate return on capital (or equivalently the household's return on equity) r_t^q increases monotonically with total factor productivity ω_t.[J.6] Accordingly, we proceed under the assumption that r_t^q increases monotonically with ω_t in our model.

Property 1. ***(Aggregate Return on Capital in a Standard NK Model)*** *In the standard NK model with capital accumulation, given a standard parametrization and a standard monetary policy rule (such as TR93 or SIT), the optimal decision rule of the aggregate return on capital (or the household's return on firm equity, r_t^q in our model) increases monotonically with total factor productivity (A_t and/or ω_t in our model).*

The dashed gray lines in Figure J.2 (panel (a)) are stylized representations of $r^q(\omega_t \mid A_t, Z_t, K_t) + \delta$ at high, intermediate, and low states of Nature. In light of Figure J.1, we represent these functions as monotonically increasing and essentially linear. The value $r^q(\omega_t = 1 \mid A_t, Z_t, K_t) + \delta$ (at $\omega_t = 1$) corresponds to the household's return on equity that prevails in the state of Nature $\{A_t, Z_t, K_t\}$ assuming that the entire capital stock is used productively (*i.e.* that $\omega_t = 1$). Recalling condition (29) for the existence of a normal–times equilibrium $\mathscr{N}$, relation (34), and Assumption 1, equilibrium $\mathscr{N}$ emerges if and only if $r^q(\omega_t = 1 \mid A_t, Z_t, K_t) \geq \bar{r}^k$. In Figure J.2 (panel (a)), normal–times equilibria $\mathscr{N}$ are represented by the green dots and emerge in the high and intermediate states of Nature. More generally, the full set of normal–times equilibria is represented by the green segment, which corresponds to all states of Nature where $r^q(\omega_t = 1 \mid A_t, Z_t, K_t) \geq \bar{r}^k$. Since we rule out coordination failures (Assumption 1) it should be clear that the existence of these normal–times equilibria $\mathscr{N}$ does not depend on the existence of a pooling equilibrium.

J.3.2 Analysis of Function $\varphi(\omega_t)$

The term on the right–hand side of condition (J.11) is a linear function of ω_t that decreases with ω_t when $\kappa \geq \frac{(1-\theta)\mu}{\theta(1-\mu)}$ and strictly increases with ω_t otherwise:

$$\varphi'(\omega_t) \leq 0 \;\text{ if }\; \kappa \geq \frac{(1-\theta)\mu}{\theta(1-\mu)} \quad \text{and} \quad \varphi'(\omega_t) > 0 \;\text{ otherwise} \tag{J.12}$$

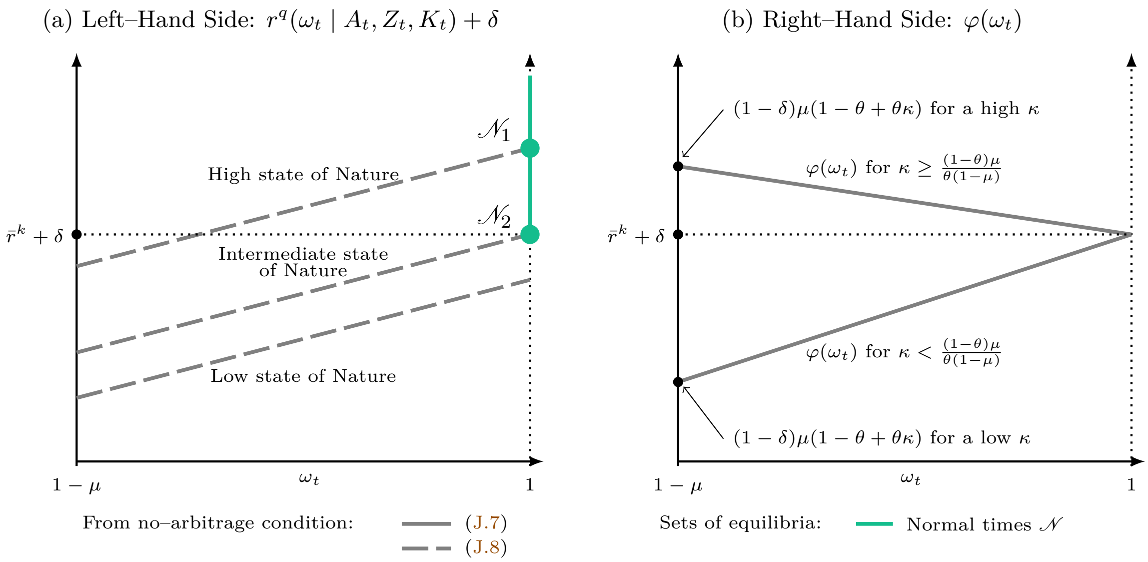
[J.5] Only in the particular case where ω_t is a white noise (row 3.), r_t^q is independent of ω_t in some states of Nature (flat lines in panels (a) and (c)).

[J.6] Recall from relation (34) that the return on equity $r_t^q = \omega_t r_t^k + (1-\omega_t)\cdot(-\delta)$ is the weighted average of productive and unproductive firms' respective returns on capital and therefore coincides with the *aggregate* return on capital.

The function $\varphi(\omega_t)$ is represented separately in Figure J.2 (panel (b)) for the case where the default cost parameter κ is high (downward–sloping line) and low (upward–sloping line).

Next, we study the existence of a general equilibrium with pooling, depending on whether the default cost parameter κ is high or low.

Figure J.2: Stylized Representation of Relation (J.11)



Notes: Panel (a): stylized representation of the term $r^q_t + \delta$ on the left–hand side of condition (J.11) as a function of ω_t, as implied by Figure J.1 and Property 1 of the standard NK model. The left–hand side of condition (J.11) comes from productive firms' no–arbitrage condition (J.8). Panel (b): representation of the term $\varphi(\omega_t)$ on the right–hand side of condition (J.11). The right–hand side of condition (J.11) comes from unproductive firms' no–arbitrage condition (J.7). $\varphi(\omega_t)$ increases linearly with ω_t if $\kappa < (1-\theta)\mu/\theta(1-\mu)$ (case with "low κ") and decreases linearly with ω_t otherwise (case with "high κ"). $\bar{r}^k + \delta \equiv (1-\theta)(1-\delta)\mu/(1-\mu)$, as defined in (29). Green line: set of normal–times equilibria, characterized by $r^q(\omega_t = 1 \mid A_t, Z_t, K_t) \geq \bar{r}^k$ (from condition (29) and relation (34)).

J.3.3 Pooling Equilibria Cannot Emerge If $\kappa \geq \frac{(1-\theta)\mu}{\theta(1-\mu)}$

When $\kappa \geq (1-\theta)\mu/\theta(1-\mu)$, $\varphi(\omega_t)$ monotonically decreases with ω_t and one can rewrite the second inequality in condition (J.5) as:

$$\omega_t < 1 \Leftrightarrow \varphi(\omega_t) \geq \varphi(1) \underset{(J.11)}{=} \frac{(1-\delta)(1-\theta)\mu}{1-\mu}$$
$$\underset{(J.11)}{\Leftrightarrow} r^q(\omega_t \mid A_t, Z_t, K_t) + \delta \geq \frac{(1-\delta)(1-\theta)\mu}{1-\mu}$$

Since $r^q(\omega_t \mid A_t, Z_t, K_t)$ increases monotonically with ω_t (see Figure J.1), $r^q(\omega_t = 1 \mid A_t, Z_t, K_t) \geq r^q(\omega_t \mid A_t, Z_t, K_t)$ for all $\omega_t \in (1-\mu, 1)$ and the above condition implies

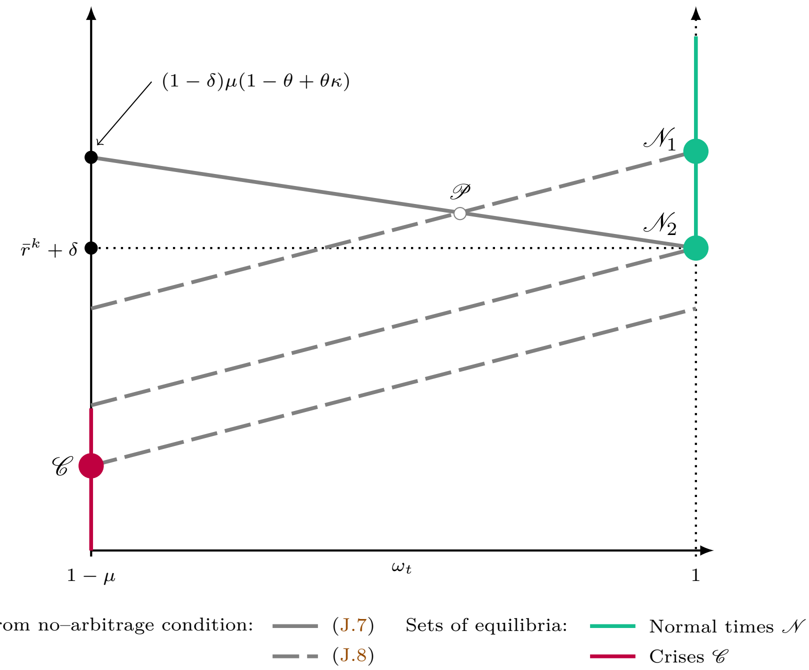
$$r^q(\omega_t = 1 \mid A_t, Z_t, K_t) \geq \frac{(1-\delta)(1-\theta)\mu}{1-\mu} - \delta \qquad \text{(J.13)}$$

which corresponds to the condition of existence of the normal–times equilibrium $\mathcal{N}$ —recall condition (29). In other words, whenever a pooling equilibrium exists, it also necessarily co–exists with $\mathcal{N}$ and can be ruled out on the grounds that it is associated with a lower value of ω_t —recall Assumption 1.

Graphical Illustration. Figure J.3 illustrates condition (J.11) in the case where the default cost parameter is relatively high, *i.e.* $\kappa \geq (1-\theta)\mu/\theta(1-\mu)$. It is constructed by superimposing panels (a) and (b) of Figure J.2 in that case, where $\varphi(\omega_t)$ *decreases* with ω_t (gray solid line). As in Figure J.1 and in Figure J.2 (panel (a)), the upward sloping dashed lines represent the function $r^q(\omega_t = 1 \mid A_t, Z_t, K_t) + \delta$ in different states of Nature. Recalling condition (29) in Proposition 2, equilibrium $\mathcal{N}$ exists whenever the value of $r^q_t + \delta$ (gray dashed line) at $\omega_t = 1$ is equal to or above the crisis threshold $\bar{r}^k + \delta$ (green dots). Since such an equilibrium always exists in the states of Nature where $r^q_t + \delta = \varphi(\omega_t)$, *i.e.* where the solid and dashed gray lines intersect, pooling equilibria cannot emerge under Assumption 1.

Figure J.3: Condition (J.11): Nonexistence of Pooling Equilibria

Functions $r^q(\omega_t \mid A_t, Z_t, K_t) + \delta$ and $\varphi(\omega_t)$: Case with $\kappa \geq \frac{(1-\theta)\mu}{\theta(1-\mu)}$



Notes: This figure is constructed by superimposing panels (a) and (b) of Figure J.2 for the case with $\kappa \geq (1-\theta)\mu/\theta(1-\mu)$. A pooling equilibrium corresponds to a point at which the solid gray line, $\varphi(\omega_t)$, crosses the dashed gray line, $r_t^q + \delta$, provided that the normal–times equilibrium $\mathscr{N}$ does not exist. A crisis equilibrium exists whenever neither a normal–times nor a pooling equilibrium exists. The figure illustrates that, under Assumption 1, pooling equilibria $\mathscr{P}$ never emerge when $\kappa \geq (1-\theta)\mu/\theta(1-\mu)$. $\bar{r}^k + \delta \equiv (1-\delta)(1-\theta)\mu/(1-\mu)$ as defined in (29).

The reason why pooling equilibria $\mathscr{P}$ always co–exist with normal–times equilibria $\mathscr{N}$ is intuitive. When default costs are high, lenders must charge a high risk premium and therefore a high loan rate as compensation for expected defaults. The higher the probability of default (the lower ω_t), the higher the loan rate that lenders must charge to cover the default costs. Hence, $\varphi(\omega_t)$ (gray solid line) decreases with ω_t. In turn, productive borrowers can only afford to pay the required loan rate in the highest states of Nature. Only in those cases can a pooling equilibrium $\mathscr{P}$ emerge. But to the extent that these states of Nature also correspond to those where equilibrium $\mathscr{N}$ exists, $\mathscr{P}$ must be ruled out on the grounds that it is associated with a lower value of ω_t compared to $\mathscr{N}$.

J.3.4 Pooling Equilibria May Emerge If $\kappa < \frac{(1-\theta)\mu}{\theta(1-\mu)}$

When $\kappa < (1-\theta)\mu/\theta(1-\mu)$, $\varphi(\omega_t)$ increases monotonically with ω_t and one can rewrite the second inequality in condition (J.5) as

$$\omega_t < 1 \Leftrightarrow \varphi(\omega_t) < \varphi(1) = \frac{(1-\delta)(1-\theta)\mu}{1-\mu} \underset{(J.11)}{\Leftrightarrow} r^q(\omega_t \mid A_t, Z_t, K_t) + \delta < \frac{(1-\delta)(1-\theta)\mu}{1-\mu}$$

Since $r^q(\omega_t \mid A_t, Z_t, K_t)$ increases monotonically with ω_t, the above condition may hold when

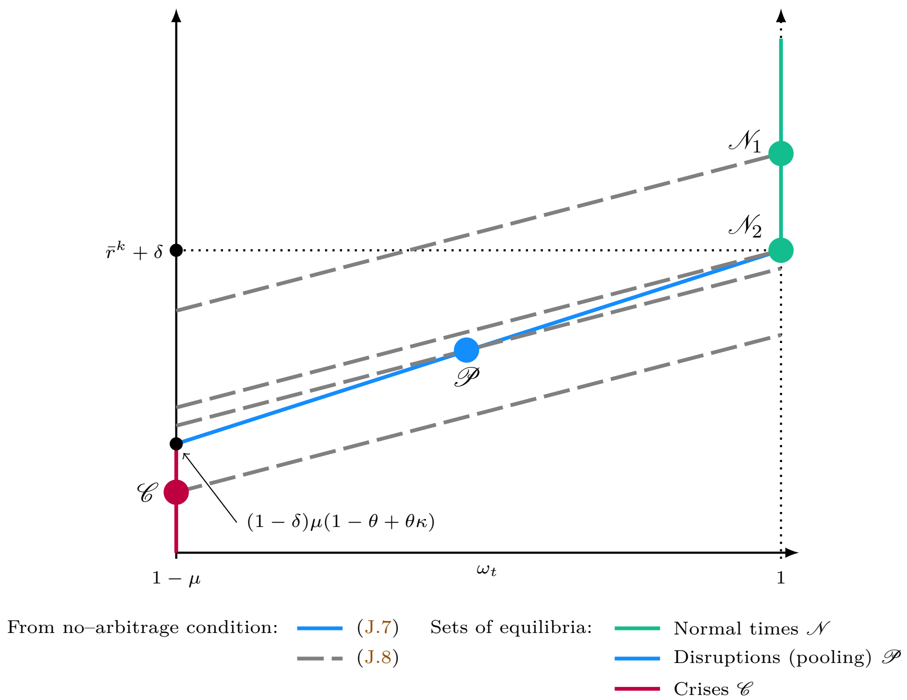
$$r^q(\omega_t = 1 \mid A_t, Z_t, K_t) < \frac{(1-\delta)(1-\theta)\mu}{1-\mu} - \delta \tag{J.14}$$

which corresponds to a case where the normal–times equilibrium $\mathscr{N}$ does not exist (from condition (29)). In that case, pooling equilibria may exist and emerge.

Graphical Illustration. Figure J.4 illustrates the condition of existence of a pooling equilibrium in this particular case where the default cost parameter κ is relatively low, *i.e.* $\kappa < (1-\theta)\mu/\theta(1-\mu)$. The figure is constructed by superimposing panels (a) and (b) of Figure J.2 in that case, where $\varphi(\omega_t)$ *increases* with ω_t and reaches a maximum equal to $\bar{r}^k + \delta$ at $\omega_t = 1$. The figure shows that pooling equilibria $\mathscr{P}$ may emerge in *intermediate* states of Nature. These equilibria correspond to situations where the dashed gray line crosses the solid blue line at $\omega_t \in (0,1)$ *and* is below $\bar{r}^k + \delta$ at $\omega_t = 1$ (*i.e.* the normal–times equilibrium $\mathscr{N}$ does not exist). In such states, lenders charge borrowers a loan rate that both compensates them for the default costs and is affordable for productive firms. When economic fundamentals are weak (the lowest dashed gray line in Figure J.4), productive firms cannot afford to pay such a loan rate (the two lines do not cross), pooling equilibria disappear, and the credit market collapses (equilibrium $\mathscr{C}$).

Figure J.4: Condition (J.11): Existence of Pooling Equilibria

Functions $r^q(\omega_t \mid A_t, Z_t, K_t) + \delta$ and $\varphi(\omega_t)$: Case with $\kappa < \frac{(1-\theta)\mu}{\theta(1-\mu)}$



Notes: This figure is constructed by superimposing panels (a) and (b) of Figure J.2 for the case with $\kappa < (1-\theta)\mu/\theta(1-\mu)$. A pooling equilibrium corresponds to a point at which the solid blue line, $\varphi(\omega_t)$, crosses the dashed gray line, $r^q_t + \delta$, provided that the normal–times equilibrium $\mathscr{N}$ does not exist. The set of pooling equilibria across all possible states of Nature (solid blue line) coincides with the upward–sloping solid gray line (*i.e.* function $\varphi(\omega_t)$) in panel (b) of Figure J.2. A crisis equilibrium exists whenever neither a normal–times nor a pooling equilibrium exists. The figure illustrates that pooling equilibria $\mathscr{P}$ may emerge when $\kappa < (1-\theta)\mu/\theta(1-\mu)$ in intermediate states of Nature. $\bar{r}^k + \delta \equiv (1-\delta)(1-\theta)\mu/(1-\mu)$ as defined in (29).

J.4 Pooling Equilibria Cannot Emerge With a Plausible Parametrization

We conclude that lenders do not tolerate defaults if $\kappa \geq (1-\theta)\mu/\theta(1-\mu)$, but may tolerate them in some states of Nature if $\kappa < (1-\theta)\mu/\theta(1-\mu)$. In the former case, pooling equilibria never emerge and the economy may be in one of only two regimes: normal times or crises. In the latter case, the economy may at times be in a third, intermediate regime of credit–market disruptions characterized by pooling equilibria and borrower defaults. Given parameters μ and θ, a *sufficient* condition to rule out pooling equilibria is

$$\kappa \geq \frac{(1-\theta)\mu}{\theta(1-\mu)} \underset{\substack{\theta=0.527 \\ \mu=0.05}}{\approx} 0.047 \tag{J.15}$$

For parameter values $\theta = 0.527$ and $\mu = 0.05$ (see Table 1), which we set to match the size and incidence of financial crises, this is the case if default costs exceed 4.7% of the debt recovery value. As discussed in Section J.2, this 4.7% threshold is much lower than lenders' debt–collection costs observed in practice. In the United States, for example, lenders often outsource the task of obtaining repayments from delinquent borrowers to third–party debt collectors, which commonly charge fees in the range of 20–50% of the amount of debt that they recover. Antill (2024) also documents that, when a US borrower files for bankruptcy under Chapter 7, the appointed trustee charges creditors substantial legal fees of about 11–13% of the debt recovery value —a lower bound for creditors' overall debt–collection cost. Based on the simulated method of moments of a dynamic macro–model fitted to Compustat data, Hennessy and Whited (2007) estimate that bankruptcy costs represent about 10.4% of the bankrupt firm's total capital stock (after depreciation) which, in the context of our model, corresponds to $\kappa = 0.104/\theta \approx 0.2$ of lenders' debt recovery value. We therefore conclude that, for plausible parameters, our model does not feature pooling equilibria. In particular, plausible default costs are too high for lenders to tolerate defaults.

Likewise, for reasonable values $\kappa = 0.12$ and $\mu = 0.05$, no pooling equilibrium can emerge if one assumes that the debt recovery rate θ is high enough, *i.e.*

$$\kappa \geq \frac{(1-\theta)\mu}{\theta(1-\mu)} \quad \Leftrightarrow \quad \theta \geq \frac{\mu}{(1-\mu)\kappa + \mu} \underset{\substack{\kappa=0.12 \\ \mu=0.05}}{\approx} 0.30$$

Assuming a recovery rate above 30% is highly plausible: for the United States, Jankowitsch et al. (2014) report an average recovery rate of defaulted corporate bonds between 40% and 70% of the bonds' face value over the period 2004–2008, while S&P Global report average recovery rates between 40% (for bonds) and 72% (for loans) over the period 1987–2023. In a model–based analysis of markets' valuation of corporate debt, Longstaff et al. (2005) assume a constant recovery rate of 50%, *i.e.* close to our baseline parameter $\theta = 0.527$.

Moreover, for reasonable values $\kappa = 0.12$ and $\theta = 0.527$, no equilibrium with defaults can emerge if the proportion μ of unproductive firms —or the productivity loss in a crisis— is not too large, *i.e.*

$$\kappa \geq \frac{(1-\theta)\mu}{\theta(1-\mu)} \quad \Leftrightarrow \quad \mu \leq \frac{\theta\kappa}{1-\theta+\theta\kappa} \underset{\substack{\theta=0.527 \\ \kappa=0.12}}{\approx} 0.12$$

In the context of our model, this means that the productivity loss that is specifically due to financial frictions should be smaller than 4.5% ($= 1-(1-0.12)^{0.36}$), which is in line with existing empirical studies (*e.g.* Oulton and Sebastiá-Barriel (2016), Gilchrist et al. (2013), Duval et al. (2019)).

More generally, it is not possible to set realistic values of the financial parameters μ, θ and κ jointly such that the necessary (but not sufficient) condition $\kappa < (1-\theta)\mu/\theta(1-\mu)$ for the existence of pooling equilibria is satisfied. Indeed, realistic values for μ must be between 0.022 and 0.065 to obtain a fall in productivity during a crisis in the range of 0.8% (Oulton and Sebastiá-Barriel (2016)) to 2.39% (Duval et al. (2019)). Moreover, realistic values for lenders' debt recovery rate θ range between 0.4 and 0.7 (Jankowitsch et al. (2014)). Since the threshold for the existence of pooling equilibria, $(1-\theta)\mu/\theta(1-\mu)$, decreases with θ and increases with μ, the highest value of this threshold is reached for $\theta = 0.4$ and $\mu = 0.065$. For these values, the threshold is equal to $(1-\theta)\mu/\theta(1-\mu) = 0.105$, which is still below the lowest plausible value of 0.12 for κ (Antill (2024)). Note that the latter result does not rest on the definition of a financial crisis, and therefore holds even if one also includes the most severe credit–market disruptions, *i.e.* those with the lowest values of ω_t, in our definition of financial crises (in addition to complete market shutdowns).

J.5 Off–Equilibrium Default Risk and Risk Premium in a Crisis

In the lowest state of Nature, a crisis is the only possible equilibrium outcome (Figures J.3 and J.4, red dots). Even though in a crisis borrowers do not default *per se*, default risk nonetheless plays an important role *off–equilibrium*. Indeed, a crisis occurs when the productive firms' return on capital is so low that firms have an incentive to borrow, abscond and default. In that case, the default risk of prospective borrowers is high and the loan rate r_t^c that lenders would need to charge to be compensated for prospective defaults if they lent (given by the no–arbitrage condition (J.7) and reflected in the solid gray line) is always above the rate r_t^k that productive firms can afford to pay (given by the no–arbitrage condition (J.8) and reflected in the lowest dashed gray line). When productive firms cannot afford to borrow, prospective lenders refuse to lend, hence a crisis. Crises in our model can thus be seen as the flip side of prohibitively high off–equilibrium default risk premia and loan rates.

J.6 Crisis Dynamics With *versus* Without Defaults

This section studies the dynamics of crises in a counterfactual economy where lenders incur small default costs and pooling equilibria arise in some states of Nature. It also compares the dynamics of this version of the model with defaults with the baseline dynamics in Figure 4. For the purpose of the comparison, we use the same parameters as in Table 1 and —to emphasize the potential differences between the two versions— focus on the extreme case where there is no default cost at all:

$$\kappa = 0$$

The statistics reported in Table J.2 show that, in the stochastic steady state, the economy with no default cost spends 11.59% of the time in a situation of credit–market disruptions (*i.e.* with $\omega_t \in (1-\mu, 1)$) and 0.35% of the time in a crisis (*i.e.* with $\omega_t = 1 - \mu$).

Table J.2: Economic Performance and Welfare: With *versus* Without Default Costs

	Time in Disruption/Crisis (in %)	Productivity Loss due to Disruption/Crisis* (in %)	Welfare Loss (in %)
$\kappa = 0$	11.59/0.35	0.5/[1.8]	0.42
Baseline ($\kappa > 0.047$)	0.00/[10]	NA/[1.8]	0.58

Notes: Same statistics as in Table 2, row (1), using the same baseline parameters as in Table 1 but assuming $\kappa = 0$ instead of $\kappa > 0.047$. *Average fall in productivity $1 - \omega_t^{0.36}$ that is specifically due to financial frictions during credit–market disruptions or financial crises. The values in square brackets are the targeted moments (see Section 5.1).

J.6.1 Dynamics Around "Abnormal" Episodes

In what follows, we refer to credit–market disruptions and credit–market shutdowns as "abnormal" episodes. Since the condition for normal times is identical in both versions of the model (refer to equation [13.] in Tables B.1 and J.1), and the periods preceding all types of abnormal episodes (whether market disruptions or shutdowns) correspond to normal times (characterized by perfect capital reallocation), it is reasonable to expect the average dynamics before abnormal episodes to mirror those before crises in our baseline. Figure J.5 shows that the dynamics before crises in our baseline model (dashed black line) and before abnormal episodes —essentially credit–market disruptions— in the general version of the model (solid red line) are indeed almost indistinguishable. In both versions of the model, abnormal episodes are preceded by a disinflationary investment and asset price boom (panels (c), (g) and (j)) as well as by a long period of low policy rates (panel (h)). The average episode occurs as the central bank raises its policy rate, causing a fall in aggregate output (panel (i)).

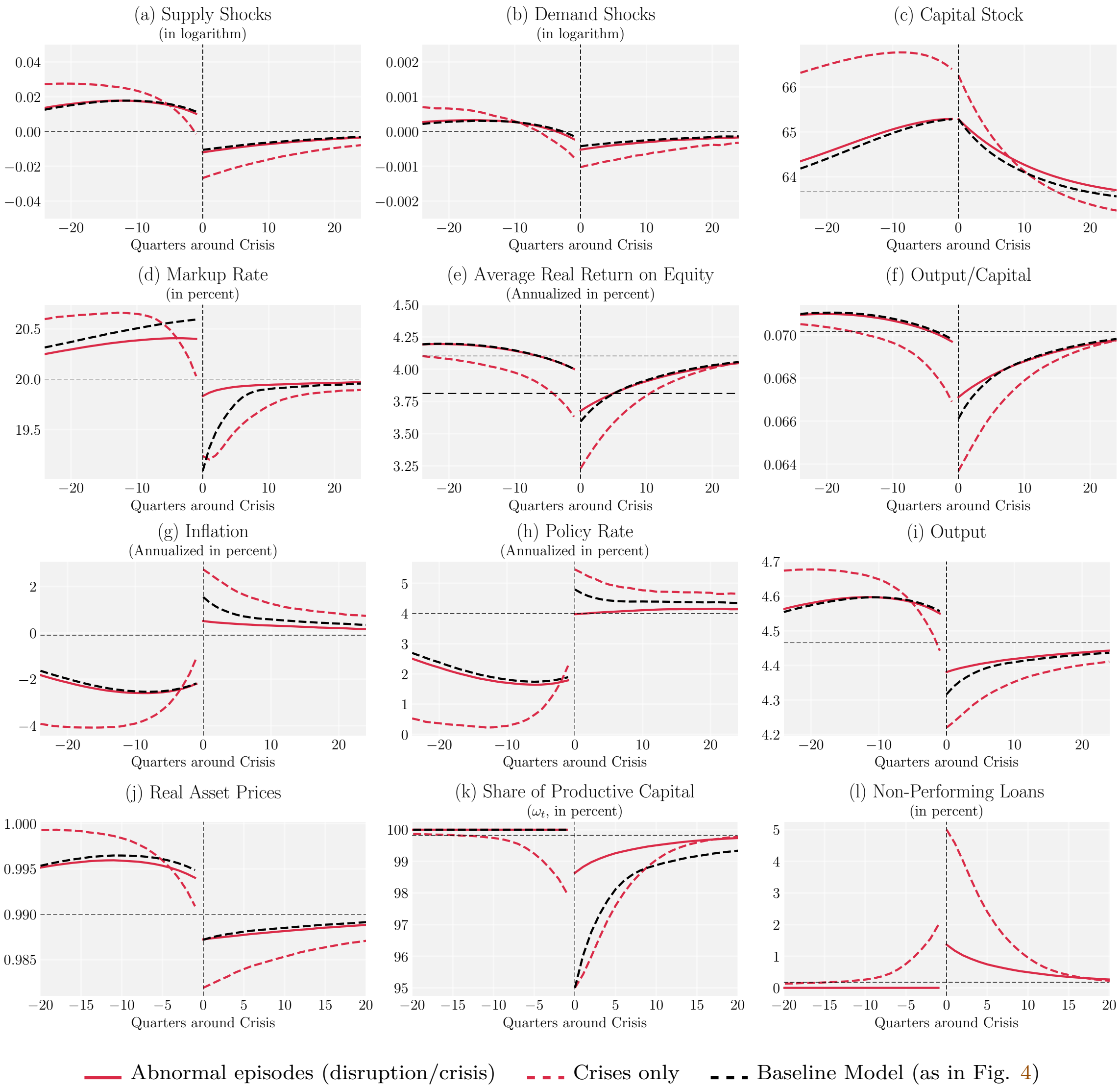
However, the dynamics in the two versions of the model differ in terms of the *aftermath* of abnormal episodes. As expected, the adverse effects of financial frictions on output are milder in the version with $\kappa = 0$ and borrower defaults because lenders continue to lend during most abnormal episodes in that version. In effect, financial frictions are milder when default costs are smaller.

During credit–market disruptions, there is less capital reallocation than in normal times but the credit market does not collapse to the same extent as during crises. Figure J.6 illustrates this point. It shows the distribution of the capital utilization rate ω_t during crises and credit–market disruptions in the stochastic steady state of the model. Given the parameters in Table 1, the economy spends 0.35% of the time in a crisis (extreme left bar), with $\omega_t = 0.95$. In that case, the productivity loss due to the collapse of the credit market amounts to $100 \times (1 - 0.95^{0.36}) = 1.8\%$ —a target of our parametrization. By contrast, credit–market disruptions feature less severe productivity losses that range from a little bit more than 0% (ω_t close to one) to 1.8% (ω_t close to 0.95). The most frequent market disruptions are the mildest ones, with ω_t close to one (extreme right bar).

In the version of the model with $\kappa = 0$ and defaults, productivity falls by 0.5% *on average* due to financial frictions during abnormal episodes (credit disruptions and shutdowns), against 1.8% during credit–market shutdowns only (Table J.2, row (1)). The fall is smaller than in our baseline model, where abnormal episodes correspond to complete market shutdowns only. Accordingly, the welfare loss relative to the first–best outcome is smaller in the general version of the model (0.42%) than in the baseline model (0.58%). Note also that, when abnormal episodes are milder (relatively more disruptions and fewer shutdowns), the capital stock tends to be on average larger and the return on capital lower in the stochastic steady state, which explains why such episodes tend to be slightly more frequent in the version of the model with defaults than in the baseline version (11.94% *versus* 10%).

Figure J.5: Simulated Dynamics Around Crises

In the Economy With Pooling *versus* Baseline

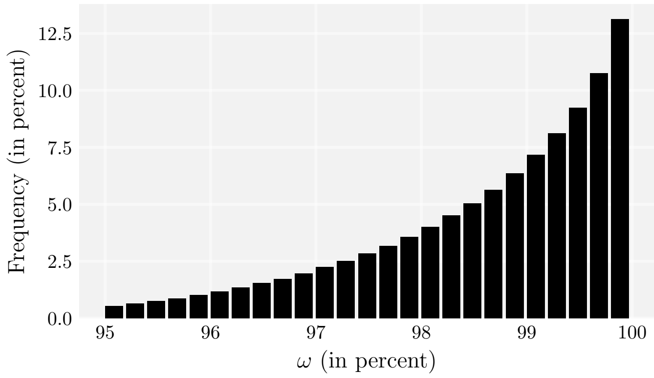


Notes: Simulations of the model with pooling for the TR93 economy. Parameters are the same as in Table 1. Abnormal episodes: either credit–market disruptions or complete credit–market shutdowns, with $1-\mu \leq \omega_t < 1$. Crises: complete credit–market shutdown, with $\omega_t = 1-\mu$. Period 0 for "Abnormal episodes": first quarter of credit disruption or crisis (*i.e.* with $\omega_t < 1$) after at least 24 consecutive quarters of no disruption or crisis. Period 0 for "Crises only": first quarter of complete credit–market shutdown (*i.e.* with $\omega_t = 1-\mu$) after at least 24 consecutive quarters without a complete shutdown (but possibly with some credit–market disruptions). Non–performing loans (panel (l)): share of borrowers that default ($1-\omega_t$).

J.6.2 Dynamics Around Crisis Episodes

The red dashed line in Figure J.5 focuses on the dynamics around *financial crises only* in the general version of the model. Crises have specific and noticeable dynamics: they occur after a few quarters of credit–market disruptions, as indicated by the preceding fall in the share of capital goods used productively (panel (k)) and the rise in non–performing loans (panel (l)). As capital mis–allocation weighs on aggregate productivity, inflation rises (panel (g)), which leads the central bank (under TR93) to increase its policy rate (panel (h)). The combination of the interest rate hike with an adverse exogenous productivity shock (panel (a)) eventually lowers the household's return on equity below the crisis threshold, triggering the complete shutdown of the credit market.

Figure J.6: Capital Utilization During Abnormal Times



Notes: Distribution of ω_t during abnormal times (credit–market disruptions as well as complete market shutdowns) in the model with $\kappa = 0$.

K Credit–Market Equilibrium Robustness: Individual Price Deviations

The aim of this section is to check that equilibria $\mathscr{N}$ and $\mathscr{C}$ are robust to price deviations from individual firms in the baseline model.[K.1] Similar to the baseline model, we assume that debt–collection costs are not unrealistically low, specifically that $\kappa > \frac{(1-\theta)\mu}{\theta(1-\mu)}$ (recall Footnote 17 in the text and the analysis in Section J of the Online Appendix). Let us consider equilibria $\mathscr{N}$ and $\mathscr{C}$ in turn.

K.1 Price Deviations from $\mathscr{N}$ Are Not Possible

In equilibrium $\mathscr{N}$, the equilibrium rate is $r_t^c = r_t^k$. For a borrower, offering a rate higher than r_t^k would lead to losses, while offering a lower rate would not attract lenders. Similarly, for a lender, offering a rate lower than r_t^k would lead to lower revenues, while offering a higher rate would not attract borrowers. Hence, price deviations from $\mathscr{N}$ are not possible.

K.2 Price Deviations from $\mathscr{C}$ Are Not Possible

In equilibrium $\mathscr{C}$, the equilibrium rate is $r_t^c = -\delta$. Assume that a given individual borrower would like to deviate from that equilibrium by offering a loan rate of $r_t^k - \varepsilon$ instead of $-\delta$, where $\varepsilon \searrow 0$ and $r_t^k = r^k(\omega_t = 1-\mu \mid A_t, Z_t, K_t)$ is the return on capital that prevails in $\mathscr{C}$.[K.2]

The loan rate $r_t^c = r_t^k$ is the highest possible loan rate that a borrower can credibly offer, since above that rate any (productive) firm that produces and (therefore) pays back its loan would make losses. Also note that assuming that $\mathscr{C}$ prevails amounts to assuming that $\mathscr{N}$ does not exist and therefore that productive firms' return on capital is below the crisis threshold, *i.e.*

$$r^k(\omega_t = 1 \mid A_t, Z_t, K_t) + \delta < \frac{\mu(1-\delta)(1-\theta)}{1-\mu} \qquad (\mathscr{C} \text{ prevails})$$

where $\omega_t = 1$ is the fraction of the capital stock that would be used productively in $\mathscr{N}$ if such an equilibrium existed (see condition (29) in Proposition 2).

The deviation of a borrower is possible only if at least one firm accepts to lend at the offered rate $r_t^c = r^k(\omega_t = 1-\mu \mid A_t, Z_t, K_t) - \varepsilon$ with $\varepsilon \searrow 0$. In turn, a firm may be willing to lend at this loan rate only if its expected payoff is higher than the payoff of keeping its capital goods idle (*i.e.* storing capital goods). Since borrowers are indistinguishable (due to asymmetric information) and since there is a probability $1-\mu$ that the deviating borrower is productive and a probability μ that it is unproductive,

[K.1] When individual firms find it optimal to adopt the equilibrium credit–market rate, it is also optimal for them to treat this rate as given, consistent with Definition 1 of the competitive equilibrium.

[K.2] The line of argument is the same if one assumes that an individual lender would like to deviate by setting its own lending rate to $r_t^k - \varepsilon$.

a firm may be willing to lend to a deviating borrower (rather than keeping its capital idle) only if:

$$\underbrace{1-\delta}_{\text{gain from storage}} \leq \underbrace{(1-\mu)(1+r_t^c)}_{\text{repaid loans}} + \underbrace{\mu\theta(1-\kappa)(1-\delta)}_{\substack{\text{payoff of the loans} \\ \text{in default, net of default costs}}}$$

which after replacing r_t^c by $r^k(\omega_t = 1-\mu \mid A_t, Z_t, K_t) - \epsilon$, setting $\epsilon = 0$, and re–arranging yields

$$(1-\mu)[r^k(\omega_t = 1-\mu \mid A_t, Z_t, K_t) + \delta] > \mu(1-\delta)(1-\theta+\theta\kappa)$$

Since we assume that $\kappa > (1-\theta)\mu/\theta(1-\mu)$, the above condition requires that:

$$r^k(\omega_t = 1-\mu \mid A_t, Z_t, K_t) + \delta > \frac{\mu(1-\delta)(1-\theta)}{(1-\mu)^2} \underset{(29)}{=} \frac{\bar{r}^k+\delta}{1-\mu} \tag{K.1}$$

$$\Leftrightarrow \underbrace{(1-\mu)\left(r^k(\omega_t = 1-\mu \mid A_t, Z_t, K_t) + \delta\right)}_{r_t^q(\omega_t = 1-\mu \mid A_t, Z_t, K_t)+\delta} > \frac{\mu(1-\delta)(1-\theta)}{1-\mu} \tag{K.2}$$

In relation (34) of the main text we establish that $r_t^q + \delta = \omega_t(r_t^k + \delta)$. In Section J of the Online Appendix, we further established that, in the NK model with capital accumulation like ours, the aggregate return on capital r_t^q increases monotonically with total factor productivity and, therefore, with the share ω_t of the economy's capital stock that is used productively (see Property 1 in Section J). As a result,

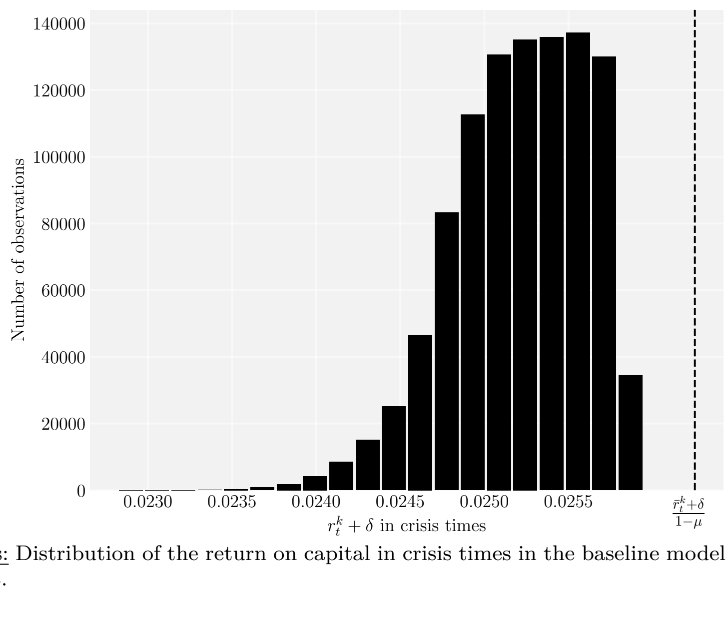
$$1 \times \left(r^k(\omega_t = 1 \mid A_t, Z_t, K_t) + \delta\right) > (1-\mu) \times \left(r^k(\omega_t = 1-\mu \mid A_t, Z_t, K_t) + \delta\right)$$

and condition (K.2) above yields the following necessary condition for a firm to accept to lend at the deviating borrower's offered loan rate:

$$r^k(\omega_t = 1 \mid A_t, Z_t, K_t) + \delta > \frac{\mu(1-\delta)(1-\theta)}{1-\mu}$$

which contradicts the assumption that equilibrium $\mathscr{C}$ prevails in the economy (see ($\mathscr{C}$ prevails)).

Figure K.1: In a Crisis, the Rate of Return on Capital is Too Low for a Borrower to Deviate



Notes: Distribution of the return on capital in crisis times in the baseline model under TR93.

In other words, if a borrower would like to deviate from the equilibrium borrowing rate when equilibrium $\mathscr{C}$ exists, its individual rate will always be too low for any lender to break even and accept to lend if $\kappa > (1-\theta)\mu/\theta(1-\mu)$. In other words, during a crisis the economic fundamentals are too weak (given the cost of a default for lenders) for productive firms to afford to pay lenders' required (break–even) rate.

One way to illustrate this result is to check numerically that in the crisis equilibrium $\mathscr{C}$ in our baseline model $r_t^k + \delta$ never fulfills condition (K.1) above. Accordingly, we report in Figure K.1 the distribution of the values taken by $r^k(\omega_t = 1-\mu \mid A_t, Z_t, K_t) + \delta$ in equilibrium $\mathscr{C}$ in the stochastic steady state of the baseline model. This figure shows that the highest loan rate that a deviating borrower can offer is *always below* the threshold required by a lender to accept to lend, *i.e.* it is always too low to cover debt–collection costs in case of default. Hence, no borrower can benefit from deviating from $\mathscr{C}$.

L Model Robustness

The aim of this section is to discuss the robustness of our results in four alternative versions of our model: (L.1) with intermediated finance, (L.3) with infinitely–lived firms, (L.2) with *ex ante* debt financing, and (L.4) with *ex ante* heterogeneous firms.

L.1 Intermediated Finance

The aim of this section is to show that our baseline model with inter–firm credit is isomorphic to a model with bank credit.

We are interested in whether capital reallocation can also take place through banks, without banks making losses. For this, we consider a representative and competitive bank that purchases K_t capital goods on credit at rate r_t^d ("deposits") from unproductive firms and sells $K_t^p - K_t > 0$ capital goods on credit ("loans") at rate r_t^ℓ to productive firms. The bank faces the same financial frictions as the firms. It is not able to enforce contracts with borrowers and does not observe firms' idiosyncratic productivities. But it is not a source of financial frictions itself, in the sense that it can credibly commit itself to paying back its deposits. The rest of the model remains unchanged.

The bank's profit is the sum of the gross returns on the loans (first term) minus the cost of deposits (last term):

$$\max_{K_t^p} (1-\mu)(1+r_t^\ell)(K_t^p - K_t) - \mu(1+r_t^d)K_t \tag{L.1.1}$$

The bank's objective is to maximize its profit with respect to K_t^p given r_t^ℓ and r_t^d, subject to its budget constraint

$$(1-\mu)(K_t^p - K_t) = \mu K_t \tag{L.1.2}$$

as well as to productive firms' participation constraint

$$r_t^\ell \leq r_t^k \tag{L.1.3}$$

and unproductive firms' incentive compatibility constraint

$$(1-\delta)K_t + (1-\theta)(1-\delta)(K_t^p - K_t) \leq (1+r_t^d)K_t \tag{L.1.4}$$

The latter constraint means that unproductive firms must be better off when they deposit their funds K_t with the bank (for a return r_t^d, on the right–hand side) than when they borrow $K_t^p - K_t$ and abscond (left–hand side).

Since the bank's profit increases with r_t^ℓ, a necessary condition for the bank to be active is that its profit be positive when r_t^ℓ satisfies (L.1.3) with equality, *i.e.* when $r_t^\ell = r_t^k$. Substituting the latter relation and relation (L.1.2) in the expression of the bank's profit in (L.1.1) yields the non–negative profit condition

$$r_t^k \geq r_t^d \tag{L.1.5}$$

Substituting (L.1.1) in (L.1.4) further yields:

$$r_t^d \geq \bar{r}^k \equiv \frac{(1-\theta)(1-\delta)\mu}{1-\mu} - \delta \tag{L.1.6}$$

The combination of (L.1.5) and (L.1.6) yields the same condition as (29) in the baseline model. It follows that, when $r_t^k < \bar{r}^k$ and the credit market has collapsed, there is also no room for financial intermediation. When $r_t^k \geq \bar{r}^k$, financial intermediation may arise. But since unproductive firms can lend to productive ones at rate $r_t^c = r_t^k$ directly *through* the credit market in that case (see equilibrium $\mathscr{N}$ in Figure 3), the bank must offer the same conditions, with $r_t^\ell = r_t^d = r_t^k$, in order to be competitive and, therefore, to make zero profit.

A version of the model with banks is therefore isomorphic to our baseline model with dis–intermediated finance. This result is intuitive. As long as banks face the same agency problem as other prospective lenders, whether financial transactions take place directly through a credit market, as in our baseline model, or indirectly through a loan market is irrelevant: these two markets rise and collapse in sync —and yield the same general equilibrium outcome.[L.1.1]

L.2 *Ex ante* Debt Financing

The aim of this section is to show that our results carry through if, at the end of period $t-1$, firms finance their startup capital stock K_t with debt instead of equity.

Assume that, at the end of period $t-1$, firms finance a share $1-\gamma$ of their capital with equity and a share γ with debt, and that debt carries a real interest rate r_t^d. In that case, a firm may end up with two distinct types of debt at the beginning of period t: a "legacy", *inter*–period debt $\gamma P_{t-1}K_t$, and a "new", *intra*–period debt $\psi_t P_t K_t$ —where ψ_t will be determined later. The implications of legacy debt issuance depend on whether firms can default on this debt or not. We study these two cases in turn.

L.2.1 Risk–free Legacy Debt

A preliminary and straightforward observation is that our model would be unchanged if we assumed that firms cannot default on households, *i.e.* that they can issue pure risk–free debt at the end of period $t-1$. This situation amounts to assuming a stronger creditor protection for legacy debtholders (households) than for new debtholders (unproductive firms).

More precisely, assume that firms incur a cost θ^h per unit of legacy debt when they hide from legacy debtholders and that this cost exceeds the gain from defaulting on legacy debt, *i.e.*

$$\theta^h \geq 1 + r_t^d \quad \forall t$$

with possibly $\theta^h = +\infty$. In that case, defaulting on legacy debt is not worthwhile, firms always repay this type of debt, and firms' incentives to default on inter–firm loans in period t are unchanged. Moreover, by the Modigliani–Miller theorem, firms would also be indifferent between financing their start–up capital with equity or debt. In such a version of the model, the capital stock K_t can therefore be seen as being entirely financed with risk–free debt (case with $\gamma = 1$) as opposed to equity (case with $\gamma = 0$).

L.2.2 Risky Legacy Debt

Next, assume that firms may default on legacy debt at the end of period t, *i.e.*

$$\theta^h < 1 + r_t^d \quad \forall t$$

As firms enter period t, lenders on the credit market understand that their legacy debt increases borrowers' incentives to default. Accordingly, they limit the amount that a borrower can borrow so that an unproductive firm does not have any incentive to borrow and abscond in normal times:

$$(1-\delta)K_t + (1-\theta)(1-\delta)(K_t^p - K_t) - \theta^h \gamma K_t \leq (1+r_t^c)K_t - (1+r_t^d)\gamma K_t$$

$$\Leftrightarrow \frac{K_t^p - K_t}{K_t} \leq \psi_t \equiv \frac{r_t^c + \delta - \gamma(1 + r_t^d - \theta^h)}{(1-\delta)(1-\theta)} \tag{L.2.1}$$

where $\theta(1-\delta)(K_t^p - K_t)$ and $\theta^h \gamma K_t$ are the costs (for the borrower) of defaulting on inter–firm and legacy debts, respectively. Relation (L.2.1) shows that market rates may have varied (and opposite) effects on incentives: a higher cost of *legacy debt* (r_t^d) deteriorates incentives (as in Stiglitz and Weiss (1981) and Mankiw (1986)), whereas a higher return on financial *assets* (r_t^c) improves incentives (as in Bernanke and Gertler (1990), Gertler and Rogoff (1990), Azariadis and Smith (1998)).

Given condition (L.2.1), the condition of existence of an active credit market becomes

$$r_t^k \geq \bar{r}_t^k \equiv \frac{(1-\theta)(1-\delta)\mu}{1-\mu} - \delta + \gamma(1 + r_t^d - \theta^h) \tag{L.2.2}$$

[L.1.1] This equivalence result only emphasizes that the key element of our model is the agency problem that lenders face, and not the financial infrastructure (financial markets or banks) considered.

where only the last term differs from the baseline condition (29). Since r_t^d is predetermined, the presence of legacy debt essentially raises the crisis threshold but does not materially affect the condition of existence of an active credit market —which still rests on the level of capital returns r_t^k.

Condition (L.2.2) illustrates and emphasizes a general result of banking models (Bernanke and Gertler (1990)): in the presence of agency costs, the equilibrium outcome is ultimately determined by the "creditworthiness" of borrowers, reflected here by the return on capital r_t^k —and not by the level of the equilibrium loan rate as such. In our model, the higher the return on capital, the more room for maneuver lenders have to address the agency problem, and the more robust the credit market.

In this extension of the model, crisis dynamics should be similar to those of our baseline model. To see this, first note that unproductive firms always default on their legacy debt during a crisis. Next, assume that households anticipate in $t-1$ a crisis in t, and therefore a higher probability of facing defaults from unproductive firms. Since households understand that debt is riskier, they will charge a higher loan rate *ex ante*: all else equal, the real loan rate r_t^d will go up. Following the increase in their cost of debt, unproductive firms' incentives to default will rise, making the crisis even more likely (condition (L.2.2)). If anything, such inter–temporal complementarities will work to amplify the dynamics in our model —rather than dampen them.

Finally, assume that firms can choose their funding mix γ at the end of period $t-1$. Given that its legacy debt impedes a firm's borrowing capacity in period t (compare (IC) and (L.2.1)), it is always optimal for this firm to finance its startup capital stock entirely through equity. Hence, if firms are given the choice of their *ex ante* debt structure, they will issue equity (which is not subject to financial frictions), *i.e.* set $\gamma = 0$ —as in our baseline model.

L.3 Infinitely–lived Firms and Stigma Effects

The aim of this section is to show that our results would not change if firms lived infinitely.

Assume that firms live infinitely, the rest of the model being unchanged —*e.g.* firms' idiosyncratic productivities are still independently distributed across periods. Since the household can freely re–balance its entire equity portfolio across firms, it is optimal for the household to perfectly diversify its portfolio and fund every firm with the same amount. Hence, all firms start afresh with the same startup equity funding and capital stock every period.

In the absence of stigma associated with default, firms' borrowing limit remains the same as in Proposition 1, and whether firms live infinitely is immaterial.

Our model is robust to introducing (some) stigma effects. Probably the simplest way to introduce stigma effects is through a penalty parameter ξ that would capture a —possibly non–pecuniary— reputational cost of default for borrowers. In that case, the payoff of a delinquent borrower would be $(1-\delta)K_t + (1-\hat{\theta})(1-\delta)K_t$ with $\hat{\theta} \equiv \theta + \xi$, and the results of the model would go through (simply replacing θ by $\hat{\theta}$).

Another way to model stigmas would be to assume that a firm that defaults is banned from the credit market for, say, $n \geq 1$ periods. In this case, the crisis threshold $\bar{r}_t^k$ would decrease with the continuation value, say V_t, of having access to the credit market in period $t+k$ with $k = 1, ..., n$. The value V_t would correspond to the discounted sum of the expected net future returns that a firm would forego by being banned from the credit market over the next n periods. It would depend, among other things, on the expectation of future rates of return r_{t+k}^k (with $k = 1, ..., n$).

While such an extension would be particularly hard to solve numerically, a simple thought experiment helps to see why the model mechanisms and results would likely not change in that case either. Consider a given crisis threshold $\bar{r}_t^k$ and an adverse exogenous productivity shock that lowers r_t^k to a level close to —but still above— $\bar{r}_t^k$. As the return on capital gets closer to the crisis threshold, firms would anticipate a higher risk of a crisis in the near future and factor in a lower franchise value V_t of having access to the credit market. Following the decline in V_t, the crisis threshold $\bar{r}_t^k$ would go up, further reducing the gap between r_t^k and $\bar{r}_t^k$. It follows that the mere expectation of a fragile credit market in the future would make the credit market collapse *even more* likely, inducing households to accumulate yet more precautionary savings. Meanwhile, intermediate good producers would further raise their markups ahead of crises (see the discussion in Section 5.3) compared to the baseline model. Such responses would lead to more frequent booms and busts. The upshot is that, in a version of the model where access to the credit market carries a franchise value that acts as borrowers' "skin in the game", the mechanisms and trade–offs would be similar to those in our baseline model.

L.4 Ex ante Heterogeneous Firms

The aim of this section is to show that our analysis and results carry through when firms are also heterogeneous *ex ante*, before they incur the idiosyncratic productivity shocks.

As an illustration, consider two observationally distinct sets of "high" (H) and "low" (L) quality firms of equal mass 1/2, characterized by probabilities μ^H and μ^L of being unproductive, with $\mu^H < \mu^L$. Households observe every firm's type H or L at the time they invest in equity and know μ^H and μ^L. But they do not observe which firms are productive within each type. The rest of the model remains unchanged.

In the presence of financial frictions, households may vary their equity investments across high and low quality firms. Let K_t^L and K_t^H denote low and high quality firms' respective initial capital stocks, with $K_t^L \neq K_t^H$.[L.4.1] The aggregate capital stock is $K_t = (K_t^H + K_t^L)/2$ and the share of K_t that is held by unproductive firms is

$$\mu_t \equiv \frac{\mu^H K_t^H + \mu^L K_t^L}{K_t^H + K_t^L} \tag{L.4.1}$$

The constant returns to scale imply that productive firms have the same realized return on capital r_t^k, irrespective of their type L or H and initial capital stock, K_t^L or K_t^H. Moreover, Proposition 1 shows that their initial capital stock does not affect firms' borrowing limit either: the borrowing limit, $(r_t^c+\delta)/(1-\theta)(1-\delta)$, is the same across high and low quality firms. Put differently, once the idiosyncratic productivity shocks are realized, what matters is whether a firm is productive, not its *ex ante* probability of being productive. It follows that the aggregate credit supply and demand schedules in normal times are given by

$$L^S(r_t^c) = \mu_t K_t$$

and

$$L^D(r_t^c) \in \left[-(1-\mu_t)K_t, \frac{(1-\mu_t)(r_t^c+\delta)}{(1-\theta)(1-\delta)}K_t\right]$$

and normal times arise in equilibrium only if there exists a credit–market rate r_t^c such that $r_t^c \leq r_t^k$ and

$$\mu_t K_t \in \left[-(1-\mu_t)K_t, \frac{(1-\mu_t)(r_t^c+\delta)}{(1-\theta)(1-\delta)}K_t\right]$$

which is the case if

$$\mu_t \leq \frac{(1-\mu_t)(r_t^k+\delta)}{(1-\theta)(1-\delta)} \Leftrightarrow r_t^k \geq \bar{r}_t^k \equiv \frac{(1-\theta)(1-\delta)\mu_t}{1-\mu_t} - \delta \tag{L.4.2}$$

The above condition is similar to (29), meaning that the Y–$\mathscr{M}$–K transmission channels of monetary policy are still present and operate in the same way as in our baseline model. The only difference is that μ_t is now endogenously determined at the end of period $t-1$, *i.e.* that the share of capital invested in low *versus* high quality firms is yet another factor affecting financial stability.[L.4.2] The upshot is that our results carry through to an economy with observationally *ex ante* heterogeneous firms, provided that there remains some residual *ex post* heterogeneity (here in the form of the idiosyncratic productivity shocks) and, therefore, a role for short–term (intra–period) credit markets.

L.5 Only One Financial Friction

Our baseline model features two standard financial frictions: moral hazard and asymmetric information between lenders and borrowers. This section shows that both frictions are needed for the aggregate equilibrium outcome to depart from the first–best outcome.

[L.4.1] One can show that it is optimal for households to hold more equity from high quality firms than from low quality firms, so that $K_t^L < K_t^H$ and μ_t varies over time. To see why, first consider the case of a frictionless credit market. Absent financial frictions, firms perfectly hedge themselves against the idiosyncratic productivity shocks and all have the same return on equity: $r_t^{q,p} = r_t^{q,u} = r_t^k$ irrespective of the realization of the shock. As a consequence, firms' quality is irrelevant and the household does not discriminate across high and low quality firms, which thus all get the same equity funding: $K_t^H = K_t^L = K_t$. Hence, $\mu_t = (\mu_H + \mu_L)/2$ and is constant over time. In the presence of financial frictions, in contrast, the household understands that unproductive firms will distribute less dividends than productive firms if a crisis breaks out. It will invest in the equity of high and low quality firms until their marginal expected returns equate and no arbitrage is possible. Since low quality firms are less likely to be productive than high quality firms and the marginal return on equity decreases with the capital stock, it is optimal for the household to invest relatively more equity in high quality firms, especially so when the probability of a crisis goes up. It follows that $K_t^H > K_t^L$ and K_t^H/K_t^L increases with the crisis probability.

[L.4.2] Since μ_t is predetermined, the effect of this additional channel can only be of second order compared to the Y–$\mathscr{M}$–K channels.

L.5.1 Asymmetric Information as Only Friction

Assume first that firms cannot abscond with the proceeds of the sale of idle capital goods. Then unproductive firms always prefer to sell their capital stock and lend the proceeds, and have no incentive to borrow. As a result, productive firms face no borrowing limit: they borrow until the marginal return on capital equals the cost of credit and $r_t^\ell = r_t^k > -\delta$ in equilibrium. No capital is ever kept idle. The economy reaches the first best.

L.5.2 Moral Hazard as Only Friction

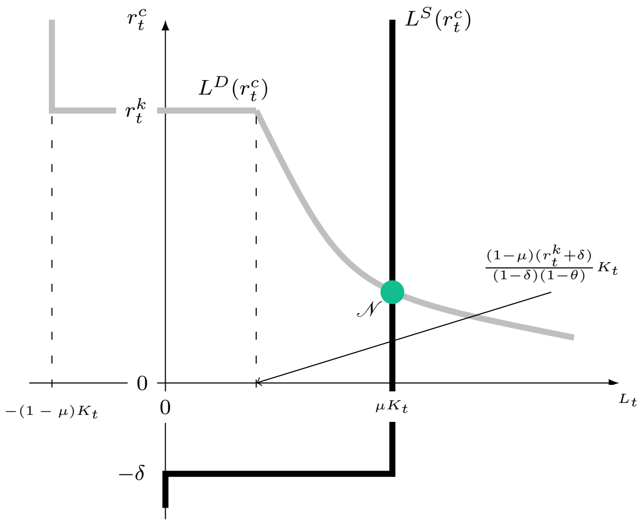
Assume next that firms' idiosyncratic productivities are perfectly observable at no cost. Then, only productive firms can borrow. But they must be dissuaded from borrowing $P_t(K_t^p - K_t)$ to purchase capital goods, keep them idle, and abscond. This will be the case if what they earn if they abscond, $P_t(1-\delta)K_t + P_t(1-\theta)(1-\delta)(K_t^p - K_t)$ is less than what they earn if they use their capital stock in production, $P_t((1+r_t^k)K_t^p - (1+r_t^c)(K_t^p - K_t))$ (from (20)), which implies:

$$(1-\delta)K_t + (1-\theta)(1-\delta)(K_t^p - K_t) \leq (1+r_t^k)K_t^p - (1+r_t^c)(K_t^p - K_t)$$

$$\Leftrightarrow \frac{K_t^p - K_t}{K_t} \leq \frac{r_t^k + \delta}{(1-\delta)(1-\theta) + r_t^c - r_t^k} \tag{L.5.1}$$

where the borrowing limit (right–hand side) now *decreases* with r_t^c: the higher the loan rate, the lower the productive firm's opportunity cost of borrowing and absconding, and hence the lower its incentive–compatible leverage.

Figure L.5.1: Credit–Market Equilibrium Under Symmetric Information



Notes: This figure illustrates unproductive firms' aggregate credit supply (black) and productive firms' aggregate credit demand (gray) curves, when credit contracts are not enforceable but information is symmetric.

The aggregate credit supply and demand schedules in Figure L.5.1 take the same form as in (24) and (25), but with the borrowing limit now given by (L.5.1) instead of Proposition 1. From Figure L.5.1 it is easy to see that there is only one equilibrium ($\mathscr{N}$) and that the economy reaches the first best: no capital is ever kept idle. The only difference with the frictionless case is in terms of the distribution of equity returns across firms: in equilibrium $\mathscr{N}$, productive firms' realized return on equity may be higher than that of unproductive firms.[L.5.1]

[L.5.1]To see this, notice that $K_t^u = 0$ in equilibrium $\mathscr{N}$, implies (from (22)) that unproductive firms' return on capital is equal to r_t^c. Further notice that $r_t^k \geq r_t^c$ and $K_t^p > K_t$ in equilibrium $\mathscr{N}$, which implies (from (20)) that productive firms' return on capital is equal to $r_t^c + (r_t^k - r_t^c)K_t^p/K_t \geq r_t^c$.

References

S. Antill (2024): "Are Bankruptcy Professional Fees Excessively High?," *Review of Financial Studies*, 37(12), 3595–3647.

C. Azariadis and B. Smith (1998): "Financial Intermediation and Regime Switching in Business Cycles," *American Economic Review*, 88(3), 516–536.

B. Bernanke and M. Gertler (1990): "Financial Fragility and Economic Performance," *Quarterly Journal of Economics*, 105(1), 87–114.

F. Boissay, F. Collard, and F. Smets (2016): "Booms and Banking Crises," *Journal of Political Economy*, 124(2), 489–538.

R. Duval, G. Hong, and Y. Timmer (2019): "Financial Frictions and the Great Productivity Slowdown," *Review of Financial Studies*, 33(2), 475–503.

J. Fernald (2015): "Productivity and Potential Output Before, During, and After the Great Recession," *NBER Macroeconomics Annual*, 29, 1–51.

M. Gertler and K. Rogoff (1990): "North-South Lending and Endogenous Domestic Capital Market Inefficiencies," *Journal of Monetary Economics*, 26, 245–66.

S. Gilchrist, J. Sim, and E. Zakrajšek (2013): "Misallocation and Financial Market Frictions: Some Direct Evidence from the Dispersion in Borrowing Costs," *Review of Economic Dynamics*, 16(1), 159–176.

G. Gorton and G. Ordoñez (2020): "Good Booms, Bad Booms," *Journal of the European Economic Association*, 18(2), 618–665.

C. Hennessy and T. Whited (2007): "How Costly is External Financing? Evidence from a Structural Estimation," *Journal of Finance*, 62(4), 1705–1745.

R. Jankowitsch, F. Nagler, and M. Subrahmanyam (2014): "The Determinants of Recovery Rates in the US Corporate Bond Market," *Journal of Financial Economics*, 114(1), 155–177.

Ò. Jordà, M. Schularick, and A. Taylor (2017): "Macrofinancial History and the New Business Cycle Facts," *NBER Macroeconomics Annual*, 31(1), 213–263.

F. Longstaff, S. Mithal, and E. Neis (2005): "Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market," *Journal of Finance*, 60(5), 2213–2253.

G. Mankiw (1986): "The Allocation of Credit and Financial Collapse," *Quarterly Journal of Economics*, 101, 455–470.

N. Oulton and M. Sebastiá-Barriel (2016): "Effects of Financial Crises on Productivity, Capital and Employment," *Review of Income and Wealth*, 63.

C. Romer and D. Romer (2017): "New Evidence on the Aftermath of Financial Crises in Advanced Countries," *American Economic Review*, 107(10), 3072–3118.

K. Rouwenhorst (1995): "Asset Pricing Implications of Equilibrium Business Cycle Models," in *Frontiers of Business Cycle Research*, ed. by T. Cooley, pp. 294–330. Princeton University Press.

J. Stiglitz and A. Weiss (1981): "Credit Rationing in Markets with Imperfect Information," *American Economic Review*, 71, 393–410.